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An Empirical Study on Basic and Conceptual Knowledge, Procedural Knowledge and Problem Solving among Primary School Students

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In this paper, we present the results of an empirical study examining the achievements of Slovenian elementary school students in arithmetic, with a particular focus on decimal numbers at the levels of basic and conceptual, procedural and problem-solving knowledge. The study aimed to determine whether there are differences or correlations between students' achievements in decimal numbers at these levels of knowledge and whether performance at one level can predict performance at another. Based on an empirical non-experimental study involving 100 Slovenian elementary school students, the findings revealed significant correlations and statistically significant differences between students' achievements at the levels of basic, conceptual, procedural and problem-solving knowledge of decimal numbers. Furthermore, performance at the levels of basic and conceptual, and procedural knowledge were found to predict performance in problem-solving tasks, and vice versa. The study's results indicate that gaps in basic and conceptual or procedural knowledge are reflected in difficulties when solving complex problems, where success often depends on the accuracy of intermediate steps within the solution process.

Keywords: decimal numbers, basic and conceptual knowledge, procedural knowledge, problem-solving knowledge, arithmetic, mathematics

INTRODUCTION

The primary objectives of mathematics education are to develop concepts and connections, acquire procedural knowledge and develop strategies that form the foundation for successful problem solving in both mathematics and everyday life. Such knowledge and skills enable individuals to engage with the system of mathematical ideas and, consequently, integrate into the culture in which they live. Mathematics is also vital for national development, as it equips students with the skills needed for

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making prudent decisions required to tackle 21st-century economic challenges (Ogbu, 2025). Elementary mathematics education focuses on fundamental mathematical concepts that are essential for all learners. These concepts are presented in ways that align with the child's cognitive development, abilities, personal characteristics and environment (e.g., nature as a resource for mathematical creativity and exploration; Žakelj et al., 2011).

Types of Knowledge in Mathematics

Taxonomies or classifications of knowledge, arranged in a hierarchical structure, provide a foundation for levels and types of knowledge. Gagné's classification (1985, as cited in Žakelj, 2003) categorises mathematical knowledge as basic and conceptual knowledge, procedural knowledge and problem solving.

Basic and Conceptual Knowledge

According to Gagné (1985, as cited in Žakelj, 2003), basic and conceptual knowledge refers to the identification and understanding of facts and concepts – categories, principles and relationships that enable individuals to organise and generalise information. Conceptual knowledge holds a significant role, as it goes beyond the memorisation of isolated facts and focuses on the ability to identify, understand and apply general principles and categories to a range of situations. Hiebert and Lefevre (1986) described basic and conceptual knowledge as the knowledge of concepts, principles and their interrelationships. Zuya (2017) further emphasised that knowledge and understanding of concepts are related to abstraction and the generalisation of particular cases. Conceptual knowledge is considered to be organised mentally in relational structures, such as semantic networks (Hiebert & Lefevre, 1986). It is not limited to a specific problem but can be generalised to a variety of problems (Schneider & Stern, 2010).

Procedural Knowledge

Procedural knowledge refers to the ability to understand and effectively execute a sequence of steps, rules or methods necessary to accomplish a task or solve a problem. It focuses on the "how-to" aspect of knowledge, emphasising the application of algorithms and procedures, techniques and strategies rather than the underlying conceptual understanding. In his taxonomy of learning outcomes, Gagné identified procedural knowledge as a critical component of cognitive skill development. Procedural knowledge is divided as follows (Gagné, 1985, as cited in Žakelj, 2003):

- Routine procedural knowledge (performing routine procedures): knowing and using rules and forms and execution of simple, routine processes, often involving straightforward problems with minimal data.

- Complex procedural knowledge (performing complex procedures): execution of complex processes requiring the understanding, flexible selection and application of more advanced algorithms and procedures (methods and processes) to solve more intricate problems involving extensive data.

Problem Solving

Gagné (1985, as cited in Žakelj, 2003) defined problem-solving knowledge as the highest level of cognitive learning in his hierarchical model of learning types. According to him, problem-solving knowledge involves the ability to apply prior knowledge and skills to solve new, unfamiliar situations or problems. It requires the integration and application of basic and conceptual, and procedural knowledge. This form of knowledge is not confined to specific tasks but also encompasses the ability to generalise and transfer knowledge to new contexts. Exploration and discovery play a critical role in this process, fostering learners' independence and flexibility in applying their knowledge (Gagné, 1985, as cited in Žakelj, 2003).

Students demonstrate problem solving in mathematics when they identify and formulate problems; assess the consistency of data; employ strategies, data and models; develop, extend and adapt procedures; apply reasoning in novel contexts; and verify the validity and effectiveness of their solutions. Problem-solving situations require students to integrate all their mathematical knowledge, including concepts, procedures, reasoning and communication skills, to address problems effectively. Moreover, solving mathematical problems leads to the acquisition of new knowledge and skills that students can apply in various situations beyond mathematics itself, as supported by numerous researchers (Freitag, 2014; Phonapichat et al., 2014; Saputro et al., 2018).

Connections Between Levels of Mathematical Knowledge

Rittle-Johnson & Alibali (1999), Suban (2012) and Al-Mutawah et al. (2019) all noted that procedural knowledge is expressed through the presentation, flexible selection and execution of algorithms, while basic and conceptual knowledge is expressed in the ability to recall definitions, rules or procedures, as well as in the understanding and representation of the meaning of mathematical concepts (Zuya, 2017). Both types of knowledge are essential for problem solving.

The development of relationships and connections between mathematical concepts occurs through meaningful learning, facilitated by cognitive activities that build relationships and connections between concepts, facts and ideas, thereby creating mental models or cognitive schemas (Suban, 2012). Different representations are used to describe concepts and their relationships (Cobb et al., 1992; Zhang, 1997), such as tables, graphs, words or symbols. These representations can be manipulated and transformed into different forms to emphasise the specific characteristics of the mathematical concepts they represent.

For example, one form of the symbolic representation of the decimal fraction 5/10 can be converted into another symbolic form: the decimal notation 0.5. Similarly, a chosen representation can be converted into a target representation (Bossé et al., 2014), such as transforming the symbolic fraction 1/2 or its decimal equivalent 0.5 into a geometric representation using a circular model. This process is referred to as the translation of a given concept.

By utilising various representations, transformations and translations and activating internal connections between concepts while integrating them with other domains, activities and problem solving, we foster understanding and give meaning to the application of mathematical knowledge (Bossé et al., 2016). Solving problem-based tasks through diverse approaches (e.g., using concrete tools, technology or analytical methods) further enhances understanding and supports the practical application of mathematical content.

The hierarchical structure of knowledge (Figure 1) shows that basic and conceptual, and procedural knowledge are fundamental components of mathematical knowledge that are essential for problem solving.

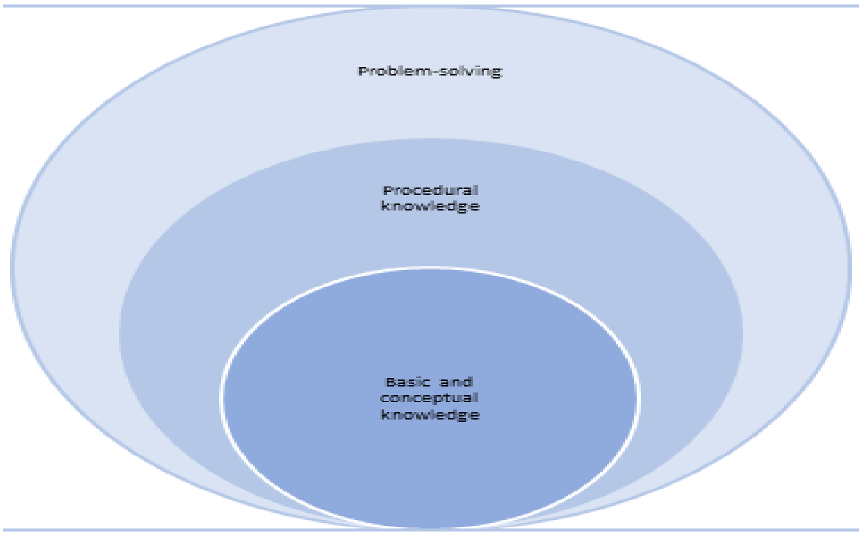

Figure 1
Basic and conceptual knowledge, procedural knowledge and problem solving

The conceptualisation of procedural knowledge and conceptual knowledge was also addressed by Baroody et al. (2007), who differentiated the qualities of basic and conceptual knowledge and procedural knowledge as superficial and deep knowledge. Superficial basic and conceptual knowledge is characterised by a weak network of connections, primarily involving foundational concepts learned within specific contexts. Deep basic and conceptual knowledge represents a strong conceptual network, encompassing both primary and secondary concepts at different levels, including abstract concepts applied across multiple contexts. Superficial procedural knowledge refers to knowledge of rules and the implementation of step-by-step procedures. Deep procedural knowledge refers to the knowledge and understanding of rules and procedures and the ability to choose and apply procedures flexibly.

In the context of this classification, Star (2005) highlighted a common misinterpretation of basic and conceptual knowledge and procedural knowledge. Specifically, basic and conceptual knowledge is often perceived as inherently complex and multifaceted, whereas procedural knowledge is frequently reduced to the execution of a procedure that is not complex in itself. According to Star (2005), the fundamental characteristic of deep procedural knowledge is flexibility. Flexibility is demonstrated by an individual's capacity to choose and adapt the most appropriate processes or procedures within a specific context (Star, 2005). Flexibility in thinking means that an individual is able to approach problems from multiple perspectives and modify problem-solving pathways when faced with impasses or cognitive obstacles (Leikin, 2007). An individual who demonstrates a high degree of flexibility will change mental pathways or solution approaches if they are ineffective and do not lead to a solution. The ability to flexibly choose procedures, including where, how and why they are applied, is critical in problem solving.

Hiebert and Lefevre (1986) and Byrnes and Wasik (1991) emphasised that mastery of both conceptual and procedural knowledge, in addition to understanding mathematical concepts and the ability to select and effectively apply procedures, enhances the capacity to detect the incorrect application of procedures. Conceptual knowledge facilitates the monitoring of mathematical operation results by providing a foundation for establishing control mechanisms to identify procedural errors (Byrnes & Wasik, 1991). For example, in the case of incorrect addition of fractions (e.g. direct addition of numerators and denominators), well-developed conceptual knowledge or understanding of rational numbers can serve as a control mechanism to point out results like 1/4 _+ 1/4 = 1/8 as incorrect or meaningless. Such knowledge emphasises that the sum cannot be smaller than the individual summand and thus points to an error in reasoning.

The knowledge domains – basic and conceptual knowledge, procedural knowledge and problem solving – are interrelated and cannot be strictly separated. In practice, we rarely rely on procedural or problem-solving knowledge alone. Instead, these domains are closely intertwined. Halford (1993) and Gelman and Williams (1997) argued that basic and conceptual knowledge and procedural knowledge cannot be developed independently; the design of procedures is based on conceptual understanding. Schneider and Stern (2010) pointed out that procedural knowledge enables fast and efficient problem solving due to its ease of automation. However, they also noted that procedural knowledge lacks the flexibility of conceptual knowledge and is often tied to certain types of problems (Baroody, 2003).

Positive correlations between basic and conceptual knowledge and procedural knowledge have been found across a variety of mathematical domains. These include counting (Dowker, 2008; LeFevre et al., 2006), addition and subtraction (Canobi & Bethune, 2008; Canobi et al., 1998; Jordan et al., 2009; Patel & Canobi, 2010), numbers and operations (Canobi & Bethune, 2008; Jordan et al., 2009) and fractions and decimal numbers (Hallett et al., 2010; Hecht, 1998; Hecht et al., 2003; Reimer & Moyer, 2005), estimation (Dowker, 2008; Star & Rittle-Johnson, 2009) and equation solving (Canobi & Bethune, 2008; Durkin, et al., 2011).

Several studies have highlighted that students' mathematical achievement in basic and conceptual knowledge tends to be lower than their achievement in procedural knowledge across content areas. Lauritzen (2012) examined students' achievement at the levels of basic and conceptual, and procedural knowledge of functions and found that a large group of students demonstrated good procedural knowledge but limited basic and conceptual understanding of functions. Students with lower performance in basic and conceptual knowledge of functions also showed lower performance in procedural knowledge in that area. However, all students who achieved a high level of conceptual knowledge of functions also demonstrated a high level of procedural knowledge. These findings highlight the interconnectedness of the types of knowledge, particularly the fundamental role of conceptual understanding in promoting procedural knowledge.

Many studies have shown that students and even adults do not have a good understanding of decimal numbers (Lai & Tsang, 2009; Moloney & Stacey, 1997; Sengul & Guldbagci, 2012). Similarly, Lai and Tsang (2009) found that while students have good procedural knowledge of decimal numbers, their conceptual understanding of decimal numbers and decimal notation is notably weak. Strong procedural performance in conjunction with weak conceptual understanding has also been noted by Al-Mutawah et al. (2019), Hong Duyen and Loc (2022) and the National External Mathematics Assessment (Republiški Izpitni Center [Slovenian National Examinations Centre], 2023).

Acording to Pulungan and Suryadi (2019), there are three kinds of learning obstacle, epistemological obstacle, didactical obstacle, and ontogenical obstacle. Each obstacle is caused by different factors. The epistemological obstacle is a limitation of students' understanding of something that is only related to a particular context according to their learning experience. The didactical obstacle is an obstacle that approaches. Ontogenical obstacle arises from student limitations, associated with neurophysiology, related to students' mental stage.

In accordance with the national mathematics curriculum, Slovenian students from Grade 2 to Grade 6 strengthen their understanding of arithmetic operations and develop foundational ideas for later algebraic learning (Russell et al., 2011; Žakelj et al., 2011). The content and objectives that 11-year-old Slovenian students are expected to achieve in arithmetic and algebra, including decimal numbers, are aligned with children's cognitive development (Žakelj et al., 2011).

However, it is important to emphasize that cognitive development does not progress at the same rate for all students. Moreover, the age of approximately 11 is typically a transitional stage during which many students move from the concrete-operational stage to formal-logical thinking. Decimals are known to be abstract numbers for students (Pramudiani et al., 2011). As also highlighted by Doz et al. (2024), understanding concepts and solving conceptual problems is often more challenging than mastering procedural knowledge or solving familiar tasks. Students frequently apply algorithms or solve problems without understanding the underlying mathematical concepts.

In the context of the ongoing curricular reform in Slovenia, which emphasizes the development of mathematical understanding and higher-order thinking skills, there is a growing need for empirical insights into how students acquire and apply conceptual and procedural knowledge. National assessment results have repeatedly revealed that Slovenian students perform better on tasks requiring procedural fluency than on those demanding conceptual reasoning. Understanding the interrelation between different types of mathematical knowledge and the factors that influence student success is therefore essential for informing instructional practices and curriculum design that effectively support meaningful learning. This study contributes to both the international and Slovenian educational discourse by providing a detailed analysis of student performance on targeted tasks and by identifying key factors that influence learning outcomes.

METHOD

Purpose and Aims of the Study

The achievements of Slovenian sixth-grade students in the 2022/23 national assessment of mathematical knowledge (Republiški Izpitni Center [Slovenian National Examinations Centre], 2023) reveal that the majority of students are capable of efficiently and reliably calculating the value of a simple numerical expression involving both decimals and natural numbers. However, only a quarter of students successfully approximate a given decimal number and represent it as a decimal fraction. Similarly, only a quarter of students are able to correctly convert a decimal fraction to a decimal number and vice versa or recognise place values in decimal numbers.

Building on the results of this national assessment and findings from numerous studies highlighting students' lower performance in basic and conceptual knowledge compared to procedural knowledge in numbers, particularly decimal numbers, this research focused on examining the relationships between different taxonomic levels of mathematical knowledge and the potential to predict performance at each level in the context of decimal numbers.

Our research focused on the achievement of Slovenian primary school students at the levels of basic and conceptual knowledge, procedural knowledge and problem solving. Specifically, we explored whether differences or correlations exist between students' achievements at these levels and whether achievements at one level can predict achievement at another.

Aims of the Study

This study aimed to determine the following:

- Students' achievements in the chosen topic of arithmetic in decimal numbers regarding basic and conceptual knowledge, procedural knowledge and problem solving

- Whether achievements in basic and conceptual knowledge, procedural knowledge and problem solving are correlated and whether there are differences between them

- Whether achievements on one or two levels can predict achievements on another level

Sample and Data Collection

The study included 100 elementary school students in the 6th Grade from six randomly selected Slovenian schools. Data were collected using three knowledge tests, encompassing a total of 90 items.

Test 1: The test comprised 10 tasks, with a total of 30 items focusing on basic and conceptual knowledge of decimal numbers. The test items included both basic knowledge (e.g., identification of place value in decimal numbers) and conceptual knowledge (e.g., justification of size relationships between decimals and abstraction across contexts). The test included an equal proportion of basic and conceptual tasks. The tasks included understanding and recognising the place values of decimal numbers and the size relationships between decimal numbers; knowing and understanding decimal fractions $\frac{a}{10^n}$, $n \in N$, and their decimal representation (writing a decimal fraction as a decimal number and vice versa); comprehending the meaning of the decimal point; rounding a decimal number to the specified position; writing and reading decimal numbers, comparing and ordering decimal numbers; determining the nearest whole number approximation of a decimal number; estimating the value of an arithmetic operation between two decimal numbers without using written algorithms and justifying the estimation; and inferring, evaluating, justifying and relating basic concepts.

Test 2: The test comprised 10 tasks, with a total of 30 items assessing students' procedural knowledge of decimal numbers. Tasks were designed to cover both routine procedural knowledge (e.g., direct computation with decimals) and complex procedural knowledge (e.g., multi-step operations embedded in word problems requiring flexible application of strategies). The test included an equal proportion of routine procedural and complex procedural tasks. The tasks required students to perform arithmetic operations with decimal numbers (e.g. written addition, multiplication and division of decimal numbers); calculate the values of numerical expressions involving decimal numbers; solve one-step word problems with decimal numbers; infer from a unit to a set; and read and interpret data from diagrams.

Test 3: This test included 10 tasks, with a total of 30 items, focusing on problem solving involving decimal numbers. The tasks required students to apply rational or decimal numbers in problem-solving contexts; select appropriate strategies for solving problems; transform a word problem into a mathematical representation (e.g. numerical expressions and equations); and analyse and draw conclusions based on data or solutions to problems.

The tests were designed in collaboration with experts in mathematics didactics, adhering to the standards of the Slovenian mathematics curriculum (Žakelj et al., 2011). Some of the items were adapted from reviews of the literature, while others were constructed by the researcher.

Table 1
Examples of tasks

Basic and Conceptual Knowledge	Procedural Knowledge
Without performing a written calculation, estimate whether the statement 8.29 + 0.99 > 10 is correct. Justify your answer.	Use a written calculation to determine how much the sum of 8.29 + 0.99 differs from 10.

Problem Solving
1. Determine the largest decimal number less than 7 with one decimal place. Multiply it by 4. Subtract from the resulting product the smallest decimal number greater than 1.44 with one decimal place. What is the final result?
2. A mother filled five jars with jam and arranged them on a shelf from smallest to largest. Each subsequent jar held 0.15 litres more than the previous jar. How many litres of jam were there in total if the largest jar held 1.1 litres?

The research followed ethical procedures. We obtained parents' consent for their children to take part in the research. The students and their parents were also informed about the objectives of the study and the data-processing protocols. Confidentiality measures were consistently implemented, and all student data were anonymised to protect student privacy. Transparency was ensured through proactive communication with all stakeholders, ensuring that all stakeholders were informed about the research process.

Data Analysis

The internal consistency (reliability) was verified using Cronbach's alpha coefficient (Field, 2005), and it was determined that reliability was very high for procedural knowledge ($\alpha prc=0.889$) and problem-solving knowledge ($\alpha prom=0.912$), and moderate for basic and conceptual knowledge ($\alpha con=0.700$).

Individual items on the test were computed according to the level of knowledge: basic and conceptual knowledge, procedural knowledge and problem-solving knowledge. Basic descriptive statistics were calculated.

Furthermore, the differences and correlations in achievements across the three levels were analysed using a paired samples t-test, and multiple regression employing the stepwise method was used to establish the possible predictors and the model design (Field, 2005).

FINDINGS

Below, we present the results of research on elementary school students in the 6th Grade on the chosen topic of arithmetic in decimal numbers.

Students' achievements in procedural, problem-solving and conceptual knowledge

Table 2
Descriptive statistics for basic and conceptual knowledge, procedural knowledge, and problem solving

	N	*Min.*	*Max.*	*M*	*SD*
Procedural knowledge	100	.09	.91	.55	.20
Basic and conceptual knowledge	100	.11	1.11	.49	.23
Problem solving	100	.00	1.23	.47	.31

N – number of students, Min – minimum, Max – maximum, M – mean, SD – standard deviation

The results (Table 2) show that the highest students' achievements were obtained for tasks designed to test procedural knowledge (M = 0.55, SD = 0.20). The average score for basic and conceptual knowledge was lower (M = 0.49, SD = 0.23). The lowest scores were obtained for problem solving (M = 0.47, SD = 0.31).

In addition, the results of paired-samples t-tests show that there was a significant difference in procedural and problem-solving knowledge (t(99) = 4.080, p = .000)), in which case students' achievements were statistically significantly higher in procedural knowledge (M = 0.55, SD = 0.20) compared to problem solving (M = 0.47, SD = 0.31). A statistically significant difference was also found between procedural knowledge and basic and conceptual knowledge (f(99) = −0.949, p = .015). Students' achievements were statistically significantly better in procedural knowledge (M = 0.55, SD = 0.20) than in basic and conceptual knowledge (M = 0.49, SD = 0.23).

The results of the study indicate that the participating students demonstrated arithmetic performance that was statistically significantly better on the selected topic of decimal numbers at the level of procedural knowledge (performing arithmetic operations with decimal numbers) compared to the level of basic and conceptual knowledge (understanding and recognising decimal numbers). Furthermore, students demonstrated performance that was statistically significantly better in procedural knowledge compared to problem-solving knowledge, where the lowest achievement levels were recorded.

Correlations and differences in students' achievements in basic and conceptual knowledge, procedural knowledge and problem solving

Table 3
Correlations among procedural, problem-solving and conceptual knowledge

	Procedural Knowledge	Problem-solving Knowledge	Conceptual Knowledge
Procedural knowledge	1		
Problem-solving knowledge	0.806**	1	
Basic and conceptual knowledge	0.474**	0.600**	1

** Correlation *is significant at the 0.01 level (two-tailed).*

The results in Table 3 and Figure 2 show that there is a statistically significant correlation between procedural knowledge and problem-solving knowledge (r = 0.806, p = .000) and between problem-solving knowledge and basic and conceptual knowledge (r = 0.600, p = 0.000). In addition, there was a statistically significant correlation between basic and conceptual knowledge and procedural knowledge, with the results indicating a moderate correlation (r = 0.474, p = 0.000).

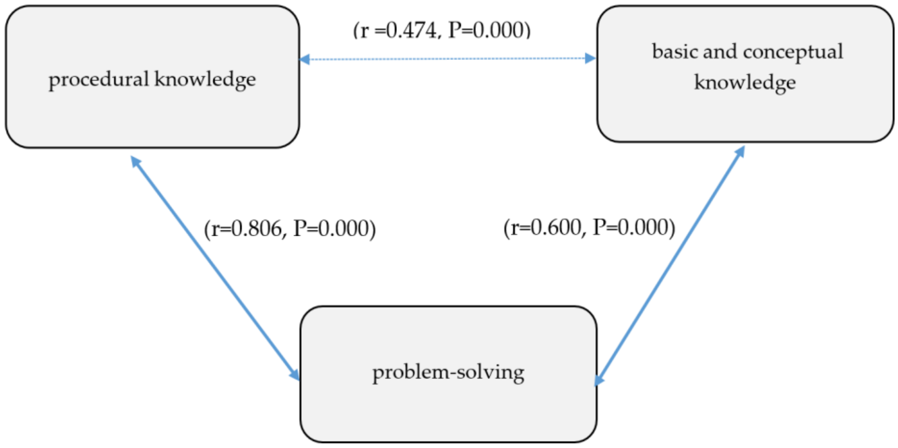

Figure 2
Correlations among conceptual knowledge, procedural knowledge and problem-solving knowledge

Predicting whether achievements on one or two levels can predict achievements on another level

A multiple linear regression was calculated to predict basic and conceptual knowledge based on problem-solving and procedural knowledge. A significant regression equation was found ($F(2, 98) = 54.985$, $p < .000$), with an R2 of 0.359. The results show that only problem solving is a significant predictor ($\beta = 0.600$, $t = 7.415$, $p =.000$) of basic and conceptual knowledge.

A multiple linear regression was calculated to predict procedural knowledge based on problem solving and basic and conceptual knowledge. A significant regression equation was found ($F(2, 97) = 89.995$, $p < .000$), with an R2 of 0.650. The results show that only problem solving is a significant predictor ($\beta = 0.806$, $t = 14.242$, $p = .000$) of procedural knowledge.

A multiple linear regression was calculated to predict problem solving based on procedural knowledge and basic and conceptual knowledge. A significant regression equation was found ($F(2, 97) = 119.030$, $p < .000$), with an R2 of 0.710. The results show that both procedural knowledge ($\beta = 0.673$, $t = 10.846$, $p = .000$) and basic and conceptual knowledge ($\beta = 0.280$, $t = 4.214$, $p = .000$) are significant predictors of problem-solving knowledge.

Based on these results, a model for predicting achievements at different knowledge levels can be designed, as shown in Figure 3 below.

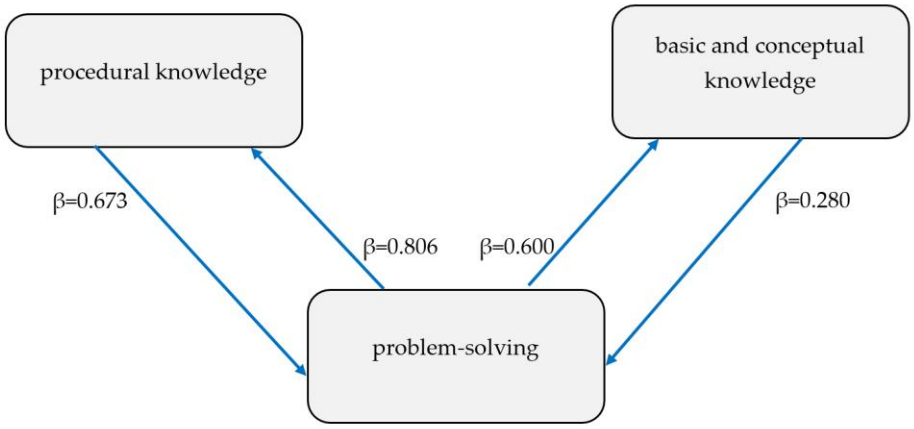

Figure 3
Regression model of procedural knowledge, conceptual knowledge and problem solving knowledge

DISCUSSION

In this section, we discuss the research results regarding the achievements of Slovenian students on a selected arithmetic topic, decimal numbers, at three taxonomic levels. The study focused on students' performance, potential differences in achievements between taxonomic levels and whether performance at one or two levels could predict performance at another level. Overall, the results revealed differences between the taxonomic levels of knowledge. Students showed proficiency in performing arithmetic operations with decimal numbers, whereas conceptual and problem-solving tasks involving decimal numbers were challenging.

Based on the results of the empirical study, we found that Slovenian students who participated in the study achieved the highest performance in procedural knowledge of decimal numbers, with performance that was statistically significantly higher compared to their performance in basic and conceptual knowledge and problem-solving knowledge.

The analysis of the results from Test 1 (basic and conceptual knowledge) revealed weak cognitive conceptual schemas regarding decimal numbers among the students included in the study. The identified difficulties included a weak ability to estimate and justify size relationships between decimal numbers, difficulties in demonstrating an understanding of place value in decimal numbers, difficulties with rounding and difficulties in identifying relevant information from word problems. The study highlighted a surprising range of misunderstandings regarding decimal numbers. In particular, students had difficulties in evaluating and justifying the value of numerical expressions involving decimal numbers, especially when multiplying and dividing decimal numbers between 0 and 1. In the research, most students were unable to justify, without using a written algorithm, whether the statement $3.28 \cdot 0.12 > 3.28$ was true ($M = 0.21$, $SD = 0.33$). More accurate estimates were observed when the multiplication tasks involved a whole number and a decimal number. For instance, when asked to evaluate and justify, without a written algorithm, whether the inequality $125 \cdot 1.1 > 125 \cdot 0.99$ holds, students demonstrated slightly higher success rates (M = 0.45. SD = 0.37).

Similarly, low student achievement at the level of conceptual knowledge of numbers has been reported in various studies (Al-Mutawah et al., 2019; Hong Duyen & Loc, 2022; Lauritzen, 2012), particularly with regard to decimal numbers, as highlighted by numerous other investigations (in Lai & Tsang, 2009; Lortie-Forgues & Siegler, 2016; Republiški Izpitni Center [Slovenian National Examinations Centre], 2023; Sengul & Guldbagci, 2012).

The research findings further revealed higher achievements in procedural knowledge compared to conceptual and problem-solving knowledge. However, even at this level, very high achievements were not observed. An analysis of the results of Test 2 (procedural knowledge) indicated that students generally did not encounter significant difficulties in performing arithmetic operations, such as addition and multiplication with decimal numbers. For example, the success rate for the task Calculate $3.28 \cdot 0.12$ in written form was reported as (M = 0.63, SD = 0.34). However, the results highlight that the mastery of procedures for all arithmetic operations remains insufficiently established. Gaps in performing arithmetic operations were particularly evident in written division with decimal numbers. For example, in the task Calculate in written form: which quotient is smaller, 125 : 0.2 or 125 : 0.25? the success rate was (M = 0.42, SD = 0.42). These findings encourage critical reflection on the interpretation of the observed disparity between students' procedural knowledge (statistically significant higher achievements) and their conceptual knowledge. This is especially true from the perspective of Gagné's taxonomy, which positions procedural knowledge above basic and conceptual knowledge. It is also noteworthy that achievements in the basic and conceptual knowledge of decimal numbers do not predict achievements in procedural knowledge and vice versa. The results indicate that problem-solving is the only significant predictor ($\beta = 0.806$, $t = 14.242$, $p = .000$) of procedural knowledge, and problem-solving is also the only significant predictor ($\beta = 0.600$, $t = 7.415$, $p = .000$) of basic and conceptual knowledge. This outcome is somewhat contradictory, since, according to Gagné's framework, basic and conceptual knowledge are the fundamental components of procedural knowledge. Moreover, the correlations between basic and conceptual knowledge and procedural knowledge were surprisingly low ($r = 0.474$, $p = .000$). Higher correlations were observed between basic and conceptual knowledge and problem-solving knowledge ($r = 0.600$, $p = .000$), as well as between procedural knowledge and problem-solving knowledge ($r = 0.806$, $p = .000$).

The findings of our research indicate that the key to understanding lies both in the underlying structure of procedural knowledge and in the pedagogical approaches employed in teaching and learning processes.

Procedural knowledge, particularly routine procedural knowledge, refers to the understanding and execution of (simple) procedures that can be acquired, to some extent, through practice and adherence to predetermined rules or step-by-step problem-solving approaches. This perspective is supported by Schneider and Stern (2010), who emphasised that procedural knowledge can be easily automated. However, procedural knowledge lacks the flexibility of conceptual knowledge and is often restricted to specific types of problems (Baroody, 2003).

We assumed that students had learned and partially mastered basic computational procedures with decimal numbers in an isolated manner, disconnected from their foundational and conceptual knowledge, without a deeper understanding (e.g. the understanding of place value in decimal numbers or the base 10 number system).

We concluded that students performed basic computational operations between decimal numbers automatically, even if they did not fully understand the magnitude of the relations between decimal numbers, place value of decimal numbers, etc. The fact that students use operations that they do not fully understand was also found by Gabriel et al. (2013) and was also evident in our research on decimal numbers.

Suban (2019) explained that in such cases, students may employ routine procedures that they do not fully understand but nonetheless perform, often uncritically. The causal relationships in learning can be summarised as 'I understand/I do not understand', 'I use/I do not use' and 'I see the purpose/I do not see the purpose.' Optimal learning occurs in the sequence I understand → I use → I see the purpose. In this scenario, during the application phase, it is expected that students will utilise acquired concepts effectively and critically. School practice also offers other situations. In a study of classroom practice regarding the application and conceptualisation of mathematical content in the context of complexity, Suban (2019) identified situations such as I do not understand → I use → ? Such situations are also indicated by the results of this research.

Similarly, Haapasalo and Kadijevich (2000) and Resnick and Omanson (1987) were unable to establish a significant connection between algorithmic procedures involving decimal numbers and the conceptual understanding of decimal numbers when investigating whether the learning of procedural knowledge is more effective when grounded in conceptual knowledge. Likewise, Lawson (2007) and Lauritzen (2012) emphasised that students often struggle with conceptual understanding, and there is increasing evidence that while students may develop procedural fluency in mathematics, they frequently encounter difficulties in achieving a deep conceptual understanding.

The lowest achievements were recorded in the problem-solving domain. An analysis of the results from Test 3 (problem-solving tasks involving decimal numbers) revealed frequent mistakes, incomplete solutions and semantically and symbolically incorrect transformations of the problem into numerical expressions or equations. In some cases, the students failed to make any transformation at all. The analysis of completed tasks indicated that these mistakes were primarily caused by a lack of familiarity with and understanding of key concepts linked to basic and conceptual knowledge. One notable example of insufficient conceptual knowledge, which subsequently impacted problem-solving success, involved a task requiring students to first identify the largest decimal number smaller than 7 with one decimal place ($M = 0.43$, $SD = 0.39$) and the smallest decimal number greater than 1.44 with one decimal place ($M = 0.30$, $SD = 0.32$). These numbers were then used in subsequent problem-solving steps. The results highlight the critical role of foundational and conceptual understanding in effective problem solving with decimal numbers. Even when students subsequently selected an appropriate problem-solving strategy, they were unable to solve the problem as a whole correctly if they had incorrectly determined the decimal numbers. The study found that students face significant challenges in understanding conceptual representations of decimal numbers, grasping the place value of decimal numbers and estimating the relative magnitudes of numbers. These difficulties negatively impact their performance in solving problem-solving tasks, similar to the challenges identified by Tambychik and Meerah (2010) and Rittle-Johnson (2017) for problem-solving tasks in general. The results also highlight that, although students' achievements in procedural knowledge with decimal numbers were the highest, the second most frequent cause of unsuccessful problem solving was insufficient mastery of arithmetic operations, particularly division. This lack of proficiency in executing basic procedures further impeded students' ability to solve complex problems effectively. The results of the study, through the analysis of achievements in problem-solving tasks involving decimal numbers, confirmed the inherent complexity of such tasks. In each phase of problem solving, basic and conceptual knowledge intersect and interact with procedural knowledge. Solving these problems requires executing multiple sequential steps, with each partial result influencing subsequent steps in the process. The causes of low achievements in problem solving were highlighted by the results of multiple linear regression analysis, which demonstrated that basic and conceptual knowledge and procedural knowledge are significant predictors of problem-solving success. Specifically, procedural knowledge ($\beta = 0.673$, $t = 10.846$, $p = .000$) and basic and conceptual knowledge ($\beta = 0.280$, $t = 4.214$, $p = 0.000$) significantly contribute to predicting students' problem-solving performance. These findings emphasise the multifaceted nature of problem-solving tasks and the critical role of integrating various types of knowledge.

The results of the study suggest that problem-solving performance with decimal numbers is strongly associated with the first and second levels of Gagné's taxonomy. The findings reveal strong correlations between basic and conceptual knowledge and problem solving ($r = 0.600$, $p = .000$), as well as between procedural knowledge and problem solving ($r = 0.806$, $p = .000$). Similar correlations between basic and conceptual knowledge, procedural knowledge and problem solving across various mathematical content areas have been highlighted in other studies. Bidirectional relationships have been found for elementary school children learning about decimal numbers (Rittle-Johnson & Koedinger, 2009; Rittle-Johnson et al., 2001). Similarly, Al-Mutawah et al. (2019) found that conceptual understanding and problem-solving skills are positively and significantly correlated with each other, according to the Pearson correlation.

The findings of the study indicate that gaps in conceptual or procedural knowledge are reflected in unsuccessful attempts to solve problem-solving tasks, which are typically complex in nature. In such tasks, individual partial results significantly influence the success of subsequent problem-solving steps. Rittle-Johnson & Alibali (1999) asserted that students possessing only procedural knowledge, due to a lack of conceptual understanding, are unable to solve real-world problems or connect concepts to problem-solving situations effectively.

The low achievement of students in problem solving was also found by educators and cognitive scientists who have agreed that fluency in recalling basic mathematical facts is essential for problem solving (Banawi et al., 2024). For the students involved in this study, gaps were identified in this area, particularly a lack of fluency in recalling and applying conceptual knowledge. In their research, Hodnik Čadež and Manfreda Kolar (2015) emphasised that an individual's ability to solve problems is closely linked to the structure of their mental schema for problem solving. The strength and compactness of this schema depend on the interconnectedness of components within and between schema groups. Such connections aid students in retaining skills and concepts and applying them appropriately when solving problems. Students with well-developed connections performed better in mathematical problem solving, whereas those with weaker connections were less successful. Similarly, Islami et al. (2018) highlighted that problem-solving success largely depends on cognitive structures, which are determined by numerous components within schema networks. Hodnik Čadež & Manfreda Kolar (2015) suggested that an individual's problem-solving ability is related to the structure of their mental schema for problem-solving, and the strength and compactness of this schema depends on the connectedness of the components between the schema groups. The links help learners remember skills and concepts and apply them appropriately to problem solving. Students with good connections are better able to solve mathematical problems, while those with poor connections are less successful. Islami et al. (2018) made a similar point: problem-solving performance is largely determined by cognitive structure, which is determined by the many components between groups of schemas in a conceptual network. According to Islami et al. (2018), mathematical connections can be classified into two groups: 1) internal connections, i.e. connections between topics and mathematical elements, and 2) external connections, i.e. connections between mathematics and other subjects, as well as between mathematics and everyday life. This means that students demonstrate and understand mathematics by making connections between mathematical concepts, facts and procedures.

School Practice

The process of developing understanding is long term and complex. It can be effectively moderated by the teacher through the thoughtful preparation of tasks and activities to support the construction and application of mathematical concepts (Suban, 2019). For effective teaching and learning of decimal numbers, it is beneficial to provide students with a variety of educational aids, such as the representation of decimal numbers on a set of beads or the representation of decimal numbers by folding paper. Such activities allow them to combine a picture of objects with a symbolic representation, thereby enhancing their ability to form abstract concepts. The aids can serve as a cognitive tool, providing scaffolding for the visualisation of concepts and relationships, supporting understanding, acting as reminders for problem-solving steps, offering reassurance in the learning process or functioning as motivational aids (Žakelj, 2014).

Sudiarta, and Suparta (2019) emphasized the benefits of the Concrete–Pictorial–Abstract (CPA) strategy in supporting conceptual understanding of fractions, an area closely related to decimal learning. Their findings highlight that students who progress through tangible and visual representations before abstraction demonstrate significantly greater understanding.

Moreover, Li (2025) explored procedural proficiency in fraction addition by comparing educational contexts in England and Taiwan. The study revealed that curriculum structure and pedagogical focus substantially affect students' performance—an insight applicable to understanding variation in decimal proficiency.

In mathematics education, teachers seem to focus more on procedural knowledge than on basic and conceptual knowledge. Such approaches to learning and teaching lead to an incorrect or incomplete understanding of mathematical concepts, as well as computational errors in procedural knowledge, as pointed out by Byrnes and Wasik (1991). Shikha and Subramaniam (2019) advised that learning and teaching decimal numbers should be built on connections to the decimal system, to measurement, to parts of a whole, to equivalent fractions, etc. A teacher's awareness of students' thinking processes is essential for planning pathways that guide students from prior knowledge to new understanding.

Doz, Cotič, and Cotič (2024) demonstrated that students' lack of understanding of fundamental mathematical concepts is often the result of omitting or shortening the concrete stage. They emphasize that the transition from concrete to abstract thinking in concept acquisition is not the objective of a single lesson or a single day, but rather a long-term educational goal. Their Slovenian-based research further shows that early grade students can achieve notable conceptual progress through problem-based instructional models—approaches that are both age-appropriate and effective.

Marentič Požarnik (2000) emphasised that cognitive conceptual networks are developed gradually and that the transition from introducing a new concept to its application in algorithms and procedures should proceed at an appropriate pace. Knowledge acquired without deeper understanding is neither enduring nor applicable (Marentič Požarnik, 2000), as illustrated by students' low achievements in problem-solving tasks involving decimal numbers. For example, computational operations between decimal numbers should be introduced when the basic concepts (e.g. place value of decimal numbers and size relationships between decimal numbers) are mastered and understood. When new knowledge is meaningfully connected to existing knowledge, the resulting understanding is of higher quality, more applicable, and longer lasting.

The research findings highlight the necessity of adopting teaching and learning approaches grounded in developmental psychology theories, which examine concept formation based on the developmental stage of children's thinking (Warren et al., 2016). These approaches should also incorporate the latest cognitive-constructivist insights in pedagogy, which emphasize the learner's active role in the learning process (e.g., Cotič & Zuljan, 2009). This implies that during the concrete operational stage, sufficient time and appropriate activities should be dedicated to supporting the development of mathematical concepts.

CONCLUSION

This study contributes to the field by providing a detailed empirical insight into the interrelation between basic and conceptual knowledge, procedural knowledge, and problem-solving skills within a well-defined curricular domain—decimal numbers. It offers an explanation for the weaker performance of students in conceptual tasks, taking into account age-appropriate cognitive development, and highlights the necessity of focusing more intensively on conceptual understanding in teaching. By doing so, it supports the design of more effective didactic strategies aimed at improving students' ability to solve complex mathematical problems and to engage meaningfully with mathematical content. The findings can also be used to provide didactical recommendations for primary school teachers who seek to support deeper understanding and the long-term development of mathematical thinking.

The findings of this study highlight the weak cognitive conceptual schemas of students involved in research concerning decimal numbers. While students demonstrated solid procedural knowledge of decimal numbers, their basic and conceptual knowledge and problem-solving knowledge about decimal numbers were significantly weaker. The weakest correlation was observed between basic and conceptual knowledge and procedural knowledge. Notably, only achievements in problem-solving knowledge were found to predict performance in basic and conceptual knowledge and procedural knowledge, and vice versa. The results also revealed that procedural knowledge of decimal numbers does not predict achievements in basic and conceptual knowledge; conversely, basic and conceptual knowledge do not predict procedural knowledge. This outcome is notable, as procedural knowledge is theoretically grounded in the conceptual schema of decimal numbers.

The results raise questions about approaches to learning and teaching decimal numbers. Mathematics teaching and learning should be based on understanding and making sense of the content. Understanding mathematical concepts and procedures is essential for successful mathematics learning. Teachers can create an appropriate environment for developing understanding by systematically and thoughtfully selecting a variety of activities that allow learners to think in a self-reflective learning environment. The results of the research highlight students' achievement in decimal numbers at three taxonomic levels, suggesting that to improve students' achievement in solving problems with decimal numbers, it is necessary to improve, in particular, their basic and conceptual knowledge of decimal numbers. The results indicate that memorising procedures (computational operations between decimal numbers) is not sufficient to develop understanding and solve problems. Therefore, it is essential to maintain a focus on understanding at all stages of the mathematics teaching and learning process. Additionally, the level of understanding among students should be continually monitored and assessed.

The findings of the study support the perspectives of mathematics didacticians who, in professional and academic discourse, emphasise that mathematics in school practice should not be viewed merely as a collection of instructions for solving problems. Instead, it should be seen as a means to stimulate and develop diverse cognitive processes, including critical thinking, creativity, the integration of digital technology and problem-solving skills. Furthermore, mathematics education should help students recognise the practical relevance and meaningfulness of learning mathematics, fostering deeper engagement and understanding.

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