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The Effect of a Training Program in Improving Academic Achievement in Mathematics

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The present study aimed at investigating the effectiveness of a training program based on mathematical thinking in mathematics on improving the academic achievement in mathematics among students of the upper basic stages in the governmental schools affiliated to Al-Salt district, Jordan. The study adopted the quasi-experimental research design that included a sample of 40 students who were randomly and equally distributed over a control group (n=20) and an experimental group (n=20). To collect data, the researcher developed a mathematical thinking skills tests (25 items) that was applied pre and post the training program. The training program based on mathematical thinking skills consisted of (6) sessions. The results revealed that there was significant statistical difference in the mean scores of mathematical thinking skills pre and post-tests among the experimental group participants who studied using the proposed training program. In addition, it was found that there was no significant statistical difference in the mean scores of mathematical thinking skills pre and post-tests among the control group participants who studied through the traditional method. Moreover, it was found that there was significant statistical difference in the mean scores of the mathematical thinking skills post-test between the experimental group who studied using the proposed training program and the control group who studied using the traditional method in favor of the experimental group. These practical implications of this study include the design of different training program that might assist in raising the level of mathematical thinking and mathematical achievement among school students at different stages.

Keywords: mathematical thinking skills, academic achievement, training program mathematics, skills

INTRODUCTION

The cognitive explosion and the rapid development and successive discoveries in the cognitive sciences led to the enormity of the scientific material and the increase in the

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courses taught to students. This cognitive explosion caused the difficulty of students storing the knowledge and information provided to them throughout the educational process (Samawi et al., 2019). Accordingly, one of the most important priorities of educators in the educational process is to resort to teaching students how to learn and think through using of methods of teaching mathematical skills to refine their thinking, develop their abilities for creative thinking, and educate them in all intellectual, psychological, spiritual, emotional, critical, decision-making and problem-solving aspects; to achieve their desired goals (Pan, Ke, & Xu, 2022)

Thinking is one of the most prominent higher cognitive mental processes behind the development of human life (Ellis, 2019). The way in which the individual thinks is a latent force in all his/her interactions, such as discovering effective solutions by which he/she overcomes the problems and difficulties he/she faces (Aminah et al., 2018). In fact, most of the scientific achievements made by mankind are based on thinking processes (Asakereh & Yousofi, 2018).

Mathematical thinking is a significant type of thinking that increases the learner's ability to understand some subjects, especially mathematics, and contributes to acquiring sound thinking methods that accompany him/her throughout his/her life (Drijvers et al., 2019). In addition, it is seen as the way that contributes to developing mathematical thinking to realize the importance of mathematical processes, abstraction, and ways of applying and developing mathematical abilities in order to understand mathematical structures (Odeh, 2016' Elastika & Dewanto, 2021).

Mathematical thinking needs a skill that can be developed through education, training, accumulation of experience, and subjecting the learner to different scientific educational situations (Jawad et al., 2021). Mathematical thinking does not happen by chance, but rather, it is necessary to develop different levels and skills of mathematical thinking among the learners, as the goal of the educational process in mathematics is to provide the learner with methods of mathematical thinking and the basics of different material such as mathematical concepts, relationships and various skills (Al Masa'fa, 2017).

In addition to realizing the importance of mathematical thinking in increasing the learner's ability to understand in mathematics and acquiring sound thinking methods that accompany him/her throughout his/her life, by presenting specific forms of thinking that are characterized as variable and powerful such as modeling, abstraction, logical analysis, inference, the use of symbols (Celik & Ozdemir, 2020' Alamri et al., 2018), and emphasizing that experience in mathematical thinking methods establish mathematical power, and increase the ability of the mind to evaluate in the era of technology, which enables the individual to critical and analytical reading, identify errors, and suggest various alternatives (Juandi et al., 2021).

Majeed et al (2021) defines mathematical thinking as an organized and flexible mental activity that aims at solving mathematical problems based on the use of some or all of the following skills: induction, deduction, expression with symbols, the process of visual-spatial perception, and formal logic and mathematical proof.

The researchers also differed among themselves in determining the most important and prominent skills of mathematical thinking, due to the different characteristics and traits of learners at each stage. Leatham (2021) mentioned four overlapping components of mathematical thinking represented by thinking as a process, knowledge, beliefs, attitudes, and the experimental research was able to identify some of the properties associated with the nature of each type of thinking.

There are many classifications associated with mathematical thinking skills, including (Morsanyi et al., 2018):

- Basic cognitive processes: which include observation, comparison, conclusion, induction, generalization, making assumptions, inference.
- Higher cognitive processes: including methods and approaches for solving problems, making judgments, innovative thinking, and critical thinking.
- Metacognitive processes: thinking for the sake of thinking, problem solving skills to include using problem solving strategies, exploring multiple approaches and solutions, and mathematical representation skills such as displaying relationships visually, symbolically, numerically, and verbally, and inference to include inductive and deductive reasoning.

While Miswanto et al (2019) reported that the mathematical thinking skills are represented by the following patterns:

- Induction to obtain a conclusion from some observations, comments, or special examples.
- Reasoning through a mature conclusion that is necessary to prove the validity of a statement, theory, or law.
- Measurement in matching a new situation with a previous one that has been proven correct, or accepting a case by based on the validity of another similar case.
- Generalization that the validity of mathematical conclusions has the same nature as generalizations of experiments in laboratory sciences.
- Deduction in obtaining a specific result based on a general principle.
- Symbolic expression through using of symbols to express mathematical ideas or verbal data.
- Recognizing relationships through the ability to extract new relationships or information that has not been previously studied.
- Mathematical proof, that is, the evidence or the argument to show that the validity of a statement stems from the validity of previous statements, through a verbal or symbolic treatment represented by a sequence of statements so that each statement is deduced from the previous one based on evidence recognized for its validity and deduction via methods approved by logic.

Mathematical thinking in mathematics subject is one of its three most prominent domains, which are the concepts, principles, and basic skills in mathematics, mathematical thinking, the nature of mathematics and the history of its development, meaning that mathematics is based on mathematical thinking and is the creative part of

the mind, so it is necessary to search for the most appropriate methods and teaching approaches to use it in the development of mathematical thinking (Niss & Højgaard, 2019). The goals related to instilling or improving mathematical ways of thinking are considered among the most prominent goals of school mathematics, as mathematics is seen as a method and pattern of thinking and an organizer for logical proof (Yunus, 2021). Mathematics curricula are a fertile field for creativity, because of their special nature and structure, which allows deducing more than one logical result for the same data and a deductive structure that gives some flexibility in organizing the content and reformulating it, and mathematics is rich in situations that can be directed to learners to find various solutions for each situation (Altakhyneh & Aburiash, 2018).

Mathematical generalizations are seen as categories of relationships between categories of concepts called rules. Therefore, to learn them at all levels of education is a general educational goal. Teachers, educators, and experts in teaching methods and curricula are working to pay attention to the achievement and development of mathematical concepts in order to understand the basics of knowledge (Kaiser, 2020).

Academic achievement is one of the topics that have attracted the attention of researchers and educators through their observations that they gathered on the varying degrees of achievement among students, which depend on the student's abilities, skills, and experience he/she possesses, and the circumstances surrounding him/her at home as the first environment in which his/her attitudes grow and develop towards learning and achievement, or the school environment as the second educational environment, by providing the school environment that is conducive and motivating for good achievement (Malik & Rizvi, 2018).

Academic achievement plays a major role in shaping and defining the learning process, given that the achievement process is complex and is influenced by multiple factors. Hence, the grades that the student obtains are not always a valid measure of his/her ability to learn, as the academic achievement process is often linked to several factors, some of which are related to the educational experience and the way of learning, and the other factors are related to the learner, his/her aptitudes, preparedness, and temperamental, emotional, and motivational traits (Darling-Hammond & Cook-Harvey, 2018)

However, Alhadabi & Karpinski (2020) reported that academic achievement is the result obtained by the student through his/her studies in previous years, that is, the sum of experiences and information obtained by the student.

Measuring the level of academic achievement aims to determine the result of the student's transition to another stage and the ability to identify the individual abilities of students, and to benefit from the results of achievement in moving from one school to another, which in turn is reflected in a either a positive or a negative way on the students' behavior towards school and education and contributes in modifying the psychological and social adjustment of students (Hwang et al., 2020).

Academic achievement includes three types, represented by the following (Peng & Kievit, 2020):

1- Academic achievement in which the student's average and performance are high compared to his/her colleagues at the same level and department, as a result of his/her use of all abilities and capabilities that ensure that he/she obtains a higher level of achievement performance expected of him/her.
2- Academic achievement in which the student's average represents half of the capabilities he/she possesses, and his/her performance is average and the degree of his/her benefit and retention of information is moderate.
3- Academic achievement in which the student's average and performance are less than the normal level compared to his/her peers. The percentage of his/her utilization and benefit from the provided course is weak, and may reach the point of non-existence.

Academic achievement is also characterized by several characteristics, including (Schreiner et al., 2020):

- It is characterized as the content of a curriculum for a particular subject or group of subjects, each with its own knowledge.
- It usually appears through the answers to the written, oral and performance semester examinations.
- Academic achievement takes care of the achievement of the majority of ordinary students in the classroom, and does not care about special features.
- Academic achievement is a collective method based on employing unified collective examinations, methods and standards in issuing evaluative judgments.

Looking at the factors affecting the level of academic achievement, they were as follows (Lei et al., 2018):

1- Student-related factors: they include his/her mental abilities, health and psychological state, and social adaptation, which in turn affect the level of individual productivity in general and the level of scientific performance of students in particular.
2- Teacher-related factors: represented by the teaching experience and his/her possession of good knowledge and appropriate teaching methods and methods, which necessarily reflects on his/her practical and educational performance.
3- Curriculum-related factors: such as clarity of objectives, the way the scientific material is presented, the way it deals with the scientific material, the extent of interaction available to the student with the content, and the ability to measure cognitive performance.
4- School-related factors: such as the classroom environment, the appropriate infrastructure in the school, and the availability of scientific facilities and laboratories.
5- Family-related factors: They include social stability, the financial situation of the family, and the educational and cultural level of the parents, because of their impact on the student's motivation and attitudes towards school and education.

Previous Studies

Gorard & Smith (2008) conducted a study to reveal the reasons for the low academic achievement in mathematics among students of the basic stage in Britain. The study

sample consisted of (2312) male and female students. The results of the study showed that the success rate of students in the mathematics subject was very low, and there were no differences in the level of achievement of students in mathematics due to the variables of gender, grade and race.

Al-Hamawi (2012) conducted a study aimed at revealing the mutual influence relationship between the self-concept and academic achievement among students of the fifth grade of basic education in the public schools of the Damascus Governorate. The study was performed over a sample of (180) male and female students. The average scores of the participants in their performance on the self-concept scale and their achievement scores, and there are differences between the average achievement scores of males and females in the sample in favor of females.

Rasheed (2015) conducted a study to identify the factors that lead to low academic achievement in mathematics among middle school students in schools in Ramadi city in Iraq from the point of view of teachers and school principals. The study sample consisted of (72) male and female teachers and (38) school principals. The results revealed that there are differences in the factors that lead to low academic achievement for middle school students in mathematics due to the variable of occupation and in favor of the teachers, and there are no differences due to the gender variable.

Al-Shalabi (2017) study aimed to find the correlation between the scientific thinking skills and the mathematical thinking skills of the sixth-grade female students. The study sample consisted of (455) female students. The study adopted the descriptive correlational approach. One of the most significant results of the study is that the level of scientific and mathematical thinking skills was moderate among the students of the basic stage, and it was clear that there was a positive correlation between the level of scientific thinking skills and the level of mathematical thinking skills. The study recommended conducting training courses for teachers to train them to employ thinking and its skills, to design scientific cases and to integrate them within the curricula and to address the weakness and deficiency in thinking skills that appear during teaching.

Faris (2017) conducted a study aimed to identify the effect of a proposed training program for the development of mathematical thinking on the achievement of the algorithm subject for a high-level technical trainee specializing in informatics/programming. The study attempted to answer the following questions: 1- What is the effect of the proposed training program on developing mathematical thinking for informatics /programming trainees? 2- What is the effect of the proposed training program for the development of mathematical thinking on the achievement of the algorithm subject? 3- Does the effect of the proposed training program on developing mathematical thinking differ according to gender? 4- Does the impact of the proposed training program on achievement in the algorithm subject differ according to gender? The study sample consisted of (64) participants, including (34) males and (30) females, studying the first semester, specializing in informatics/programming, divided into two groups (experimental and control). The researcher used the following tools: a Mathematical thinking test, a proposed program for developing mathematical thinking and an algorithm test. The experimental group was subjected to the proposed training

program for the development of mathematical thinking in its eight domains: induction, deduction, guesswork, symbolic expression, modeling, logical thinking, generalization and inference. The results indicated the positive impact of the proposed training program for the development of mathematical thinking on the achievement of the algorithm subject for all participants, males and females alike.

Rizk (2018) conducted a study to find out the effectiveness of using real learning methods in developing mathematical thinking skills (induction, deduction, expression with symbols, formal logic, and mathematical proof) for first-intermediate graders in the middle school in Makkah Al-Mukarramah. A random sample of (62) students from the first-intermediate grade in the city of Makkah Al-Mukarramah. The study found that there are differences between the mean scores of the experimental group and the mean scores of the control group in favor of the experimental group in the post-measurement of mathematical thinking skills (induction, deduction, expression with symbols, formal logic, and mathematical proof). The study recommended holding training courses for middle school teachers on real learning in teaching mathematics.

Al-Fadli & Abu Loom (2019) carried out a study aimed to reveal the effect of a proposed teaching program in developing the mathematical thinking skills of kindergarten students in the State of Kuwait. The study followed the quasi-experimental approach, the study sample consisted of two experimental groups of (25) male and female students who were purposefully selected from government kindergartens in the Department of Education and a female student, and a control group of 25 (students) in the capital in the State of Kuwait. To achieve the objectives of the study, a program was developed The results of the study showed that there is a statistically significant difference between the mean scores of the experimental group that was taught using the teaching program and the average scores of the control group students who studied in the usual way on the test of mathematical thinking and in favor of the experimental group. The researcher made a set of recommendations, including: the use of the teaching program by kindergarten teachers, and its circulation to government kindergartens affiliated with the Ministry of Education in the State of Kuwait

Commenting on the previous studies

There have been numerous studies related to mathematical thinking skills and the level of academic achievement, as a number of these studies were conducted at the level of students for the basic stage, such as the study of Al-Shalabi (2017) to find out whether there is a correlation between scientific thinking skills and mathematical thinking skills among sixth-grade students or not, the study of Al-Hamawi (2012) to reveal the mutual influence relationship between the self-concept and academic achievement among students of the fifth grade of basic education in the public schools of Damascus Governorate, and the study of Gorard & Smith (2008) to reveal the reasons leading to low academic achievement in mathematics among students of the basic stage in Britain, whereas others were conducted at the level of the upper basic stage (intermediate), such as the study of Rizk (2018) to find out the effectiveness of using real learning methods in developing the mathematical thinking skills of first-intermediate grade students in the middle school in Makkah Al-Mukarramah, and the study of Rashid (2015) to identify

the factors that lead to low academic achievement in mathematics among middle school students in the schools of Ramadi city in Iraqi of, and benefited from it in determining the size of the study sample and interpretation of results.

Research Problem and Questions

Despite the importance of thinking in general in an individual's life, there is a special importance for thinking skills, as it represents the tool through which he/she is able to face the problems and life variables that confront him/her, and helps him/her gain knowledge to extend that to his/her personal life and also enables him/her to resolve the contradictions around him/her.

The problem of the study stems from the lack of skills related to mathematical thinking among students of the upper basic stage, the interest of many mathematics teachers in the cognitive aspect and the lowest level of thinking, which is the level of memory only, and the lack of interest or care for the main goal of mathematics teaching, which is mathematical thinking, and the lack of teaching methods and means used in teaching mathematics to students about meeting the needs and characteristics of these students in such an educational stage based on the researcher's observation, where the traditional methods of teaching focus on memorization and indoctrination, and move away from the main objective of the subject in general, which is to develop students' mathematical thinking skills.

The researcher also chose the primary stage to conduct this study because of the great importance of this stage in the lives of individuals, and its distinction from other stages of study. Accordingly, this study came to answer important questions whose results provide a clear diagnosis of the impact of a training program based on mathematical thinking skills in raising the level of academic achievement in mathematics for students of the upper basic stage in public schools affiliated to Qasaba Salt, and to raise their abilities and skills. The study questions are formulated as following:

1- Are there statistically significant differences between the average scores of the experimental group students who study mathematics according to the proposed training program in the mathematical thinking skills pre and post-test?
2- Are there statistically significant differences between the average scores of the control group students who study mathematics according to the traditional method in the mathematical thinking skills pre and post-test?
3- Are there statistically significant differences between the average scores of the experimental group students who study mathematics according to the proposed training program, and the average scores of the control group students who study the same subject according to the traditional method in the mathematical thinking skills pre and post-test?

Significance of the study

The significance of the study lies in the fact that it may benefit teachers and those in charge of mathematics curricula in identifying the level of mathematical thinking skills among students of the upper basic stage in mathematics, giving them the opportunity to

develop and invest them in the process of education, organization and presentation of the academic content on foundations based on the levels of mathematical thinking, due to its significant impact on the disparity in academic achievement among students of the upper basic stage in mathematics. In addition, this study may add a new and effective method to the teaching methods and approaches that are aiding the teacher, which helps him/her to perform his/her task with high efficiency, which helps to control learning, training and exercise in performing mathematical operations and clarifying concepts so that it has a positive effect on the level of academic achievement

Moreover, it is hoped that a wide range of services will be developed for teachers and students and an appropriate environment will be chosen that achieves these goals and objectives, and that using it as an educational method provides special attention to each student according to his/her abilities, preparations and scientific level.

Further, this study may provide the developers of mathematics curricula with contemporary issues that help them choose the appropriate content to face the present and address challenges, thus opening the way for researchers to develop and reformulate the content. Furthermore, this study will be paving the way for researchers and scholars to conduct future studies related to the subject of the study, relying on the findings of the current study and the recommendations it will provide. Finally, this study will contribute in expanding the tests used and related to them in the Jordanian context, and presenting some suggestions and recommendations based on the findings of the current study.

Research Objectives

The current study aimed to reveal the effect of a training program based on mathematical thinking skills in raising the level of achievement in mathematics for students of the upper basic stage in government schools affiliated to Al-Salt District.

Conceptual and Operational Definitions

First: Mathematical thinking skills: a type of thinking that requires deduction and deep thinking about mathematical ideas that are not fully available through the five senses (Ibrahim, Hajjaj & Amin, 2017).

Procedurally: the degree obtained by the study participants through their response on the mathematical thinking skills test developed by the researcher.

Second: The level of academic achievement: the degree of acquisition achieved by the student or the level of success he/she achieves or reaches in a specific subject or educational or training field (Ritchie, 2018).

Procedurally: it is the score obtained by the study participants on the items related to measuring the level of academic achievement used in the current study.

Third: The training program: It is a set of educational activities, means and methods, used to provide both the mathematical content and assessment questions for a unit of study in mathematics, with the aim of developing mathematical thinking skills and raising the level of academic achievement among students of the upper basic stage.

The Study Limitations

The generalization of the results depends on the characteristics of the sample and the degree of its representation of the population from which it is drawn, the study tools, its domains and characteristics.

METHOD

In this study, the researcher adopted the quasi-experimental approach based on examining the effect of the training program as an independent variable on mathematical thinking skills as the dependent variable, to examine the effect of the training program based on mathematical thinking skills in raising the level of academic achievement in mathematics among students of the upper basic stage in governmental schools affiliated to Qasaba Salt. Table (1) shows the design of the study in symbols.

Table 1
Design of the study

Groups		Pre-test	Intervention	Post-test
EG1	Experimental group	O1 O2	The training program	O1 O2
CG1	The control group	O1 O2	No training program	O1 O2

Research population and sample

The study population consisted of all students of the upper basic stage in governmental schools affiliated to Al-Salt District Directorate of Education, Jordan, whose number is (2698) male and female students in (18) governmental schools.

Regarding the sample of the study, it was recruited through the purposeful random method. The study sample consisted of (40) male and female students, of whom (20) male and female students were for the control group, and (20) male and female students for the experimental group. The sample was selected purposefully after coordinating with the school principal and the mathematics teacher in the school. The researcher also administered the academic achievement scale to the study sample who obtained low scores in the scale. Table (2) below shows the distribution of the study population and its sample.

Table 2

Distribution of the study population and sample based on the study variables Variable	Categories of the variable	Study sample	Percentage
Gender	Male	21	52
	Female	19	28
Grades	Seventh	10	25
	Eighth	10	25
	Ninth	10	25
	Tenth	106	25
Total		**40**	**%100**

The data collection instrument

The researcher developed a scale to measure the level of academic achievement of the students of the upper basic stage in mathematics, consisting of (25) items. The scale was

developed based on reviewing the relevant literature (Al-Hamwi, 2010; Najem, 2012; Rashid, 2015), as well as relying on the theoretical background in the light of the definition of the level of academic achievement, and seeking the opinions of the referees who were experts in mathematics education and mathematics curriculum design. The scale items were divided into five domains:

- Student-related factors (5 items).
- Teacher-related factors (5 items).
- Curriculum-related factors (5 items).
- School-related factors (5 items).
- Family-related factors (5 items).

Each item in the previous domains was scaled using 5-point Likert scale, as following: Strongly agree (5), agree (4), neutral (3), disagree (2), strongly disagree (1), for positive items, and vice versa for negative items.

The Face validity

The scale was submitted to (5) arbitrators specialized in educational sciences, curricula and teaching methods, and specialists in measurement and evaluation. They were asked to express their opinions on the validity (appropriateness to achieve the research objectives, linguistic formulation and suitability to the research setting) and clarity of the items to measure the level of academic achievement of the students of the upper basic stage. The arbitrators agreed on all the items of the scale with minor linguistic modifications.

Second: Internal Consistency validity

Items correlation coefficients with the total scale were calculated to detect the internal consistency of the scale on a pilot sample of (20) male and female students from the study population who were excluded from the original study sample. The correlation coefficients of the items with the tool as a whole ranged between (0.453-0.756) as shown in table (3).

Table 3
Correlation coefficient matrix between the scale items and the total score

Item	Correlation with total score	Item	Correlation with total score	Item	Correlation with total score
1	.533**	10	.439*	19	.443*
2	.420*	11	.756**	20	.493*
3	.428*	12	.698**	21	.679**
4	.470**	13	.583**	22	.472*
5	.587**	14	.541**	23	.572**
6	.597**	15	.632**	24	.492*
7	.454*	16	.489*	25	.498*
8	.562**	17	.552**	26	.702**
9	.719**	18	.479*		

*Statistically significant at significance level (0.05) **Statistically significant at significance level (0.01).

It might be noticed from Table (3) that the correlation coefficients of the items with the total score were statistically significant at the significance levels α = (0.05) and (0.01), so none of them were deleted, which indicates that the scale is appropriate for measuring the level of academic achievement among students of the upper basic stage. It has high validity and is appropriate for the purposes of the current study.

Third: Scale reliability

The reliability coefficient was used according to Cronbach's alpha equation, and the test-retest reliability of the tool was used for the purposes of verifying the scale reliability. The total internal consistency coefficient by the Cronbach alpha method was (0.87), while it was (0.91) through the test-retest method.

The training program based on the mathematical thinking skills

The training program aimed at developing the mathematical thinking skills of the upper basic stage students participating in this program along with the mathematical knowledge it provides. Where mathematical thinking was defined with a number of thinking skills, which the researcher obtained through reviewing educational theoretical literature from previous books and studies that dealt with mathematical thinking and its various skills, in addition to benefiting from the mathematics curriculum and its main headings in the primary education stage, with its general and private objectives that are linked to the development of mathematical thinking among students under study.

The mathematical content was presented through activities, means and methods such as linking new learning with previous learning, using the exploratory method in presenting the educational material, presenting the mathematical topics as a coherent and connected series with each other, and presenting exciting and interesting life situations and applications for the student, making him/her feel that mathematics is useful and connected to his/her daily life, as well as achieving linkage and integration between the mathematics curriculum and the curricula of other school subjects.

The training program also included the introduction of a set of assessment questions (classroom exercises, drills and questions) through the following training activities and methods:

1- Inductive thinking: Write the generalization (in words or symbols) that you derive from the following problem (2/5 x 5/2 = 1). The generalization was that the product of the rational number (except zero) by its reciprocal equals (1).

2- Deductive thinking: If we assume the following mathematical generalization: (The rational number is the number that can be written in the form x / y, where x, y are integers, y ≠ zero, and accordingly we can conclude that all rational numbers are integer and positive.

3- Logical thinking: If we suppose the following rule: If a rational number appears in the upper half, the inverse of that number appears in the lower half. Therefore, if the rational number (5/7) is written in the upper half, the inverse of that number appears, which is (7/5) in the lower.

Accordingly, a program consisting of (6) sessions was developed, the duration of each session (60) minutes, and it was applied in a special center. The following is a presentation of the sessions of the program:

The first session (introduction and welcoming) included providing the space for acquaintance between the participating students and the researcher, introducing the program and its objectives and what can be achieved through it, with the aim of discussing the participating students, clarifying and discussing the mechanism of implementing the program and arranging appointments and sessions for the training program.

The second session (Mathematical thinking skills, its concept, importance, and types): This session included giving a clear and comprehensive conception of the concept of mathematical thinking skills in general, identifying its importance and patterns, and examining the effects of developing such skills, by providing students with the skill in conducting processes upon which mathematical thinking is based.

The third session (inductive thinking, its concept, linking it to mathematics): It included giving a clear perception of the concept of induction in general, linking it to mathematics, giving clear examples, as well as studying the relationship between a number of individual cases in a particular situation, and revealing the basic common characteristic between these individual cases, then formulate this common characteristic into a general rule.

The fourth session (deductive thinking, its concept, linking it to mathematics): It included giving a clear perception of the concept of deduction in general, linking it to mathematics, giving clear examples, and clarifying the extent of the relationship between the general rule and its special cases that can fall within it, and that what is true over the rule is valid for its individual parts and cases, and then the exact interpretation of the meaning of the general rule.

Fifth session (logical thinking, its definition, linking it to mathematics): It included giving a clear perception of the concept of logical thinking in general, linking it to mathematics, and giving clear examples to enhance students' abilities to move from the known to the unknown guided by objective rules and data.

The sixth session (finishing and evaluating the training program): It included a discussion of the group members with the pros and cons of the program, the amount of improvement that the group members feel after completing the sessions, identifying the strengths and weaknesses, and in the end thanking the members of the group participating in the program.

Groups' equivalence and homogeneity

To verify the equivalence of the groups, the mean scores and standard deviations of students' performance on the mathematical thinking skills scale were extracted according to the group variable (experimental, control), using the t-test because data normality assumption was achieved. The results shown in table (3) illustrate the mathematical thinking skills pre-test results.

Table 4
Means and standard deviations scores for the students' performance on the mathematical thinking skills pre-test based on group variable (experimental, control)

	Group	M	SD	n	t	Sig
Mathematical thinking skills	Experimental	3.76	.494	20	.476	.738
	Control	3.85	.348	20		

The results presented in table (4) shows that there are no statistically significant differences at the significance level (0.05) referred to the group variable in the total score of the mathematical thinking skills pre-test, and this result indicates the equivalence and homogeneity of the study groups.

FINDINGS

The first question: Is there a statistically significant difference at the level of significance (0.05) in the mathematical thinking skills pre and post-tests between the mean scores of the experimental group students who study mathematics according to the proposed training program?

Table 5
Paired sample t-test to compare the mean scores of the mathematical thinking skills pre and post-tests for the experimental group members

Experimental group	n	M	SD	df	t		Sig
					Calculated	Tabulated	
Pre	20	17.838	3.586	38	8.291	2.07	≤0.05
Post	20	25.729	2.824				

It is evident from Table (5) that the mean scores of the experimental group students in the mathematical thinking skills pre-test were (17,838), with a standard deviation of (3.586), while the post-test mean score was (25,729) with a standard deviation (2.824), while the calculated t-value for the paired samples was (8.291), which is greater than the tabulated value of (2.07) with a degree of freedom (38) and at a level of significance (0.05), which indicated that there was a statistically significant difference at a level of significance (0.05) in the mathematical thinking skills pre and post-tests between the mean scores of the experimental group students who are studying mathematics subject according to the proposed training program.

In order to measure the effect size and in order to confirm that result, the researcher calculated the practical significance of the results through using Eta square value ($\eta 2$), which is used to determine the degree of importance of the result whose existence was statistically proven, and its value was (0.873), which ensured the existence of an important and significant effect of the independent variable the training program in the dependent variable (mathematical thinking skills) for the students of the experimental group.

The second question: Is there a statistically significant difference at the level of significance (0.05) in the mathematical thinking skills pre and post-tests between the mean scores of the control group students who study mathematics according to the traditional method?

Table 6
Paired sample t-test to compare the mean scores of the mathematical thinking skills pre and post-tests for the control group members

Control group	n	M	SD	df	t		Sig
					Calculated	Tabulated	
Pre	20	16.812	3.826				0.2459
Post	20	18.053	2.745	38	0.692	2.06	

The results shown in table (6) indicated that the mean score of the students of the control group in the mathematical thinking skills pre-test was (16,812) with a standard deviation of (3.826), while the mean score of the students of the same group in the post-test was (18,053) with a standard deviation of (2.745), and that the calculated t-value (0.692), which is less than the tabulated value of (2.06) with a degree of freedom (38) and at a significance level (0.05), which indicates that there is no statistically significant difference at a significance level (0.05) between the mathematical thinking skills pre and post-tests mean scores of the students of the control group who studied mathematics through the traditional method.

The third question: Is there a statistically significant difference at the level of significance (0.05) in the mathematical thinking skills post-test between the mean scores of the experimental group students who study mathematics according to the proposed training program and the mean scores of the control group students who study the same subject according to the traditional method?

Table 7
Independent samples t-test for the differences in the mathematical thinking skills post-test between the experimental group and the control group

Post-test	n	M	SD	df	t		Sig
					Calculated	Tabulated	
Experimental	20	25.671	2.967	38	8.143	2.02	≤0.05
Control	20	19.827	2.639				

The results presented in table (7) showed that the mean score of the experimental group students in the mathematical thinking skills post-test was (25,671) with a standard deviation of (2.967), while the mean score of the control group students was (19.827) with a standard deviation of (2.639) and the calculated t-value (8.143) , which is greater than the tabulated value of (2.02) with a degree of freedom (38) and at a significance level (0.05), which indicated a difference between the mean score of the experimental group students who study mathematics according to the proposed training program, and the mean score of the control group students who study the same subject according to the traditional method in the mathematical thinking skills post-test, and this means that teaching using the proposed training program was more effective in developing mathematical thinking skills for students of the upper basic stage in mathematics compared to teaching in the traditional method.

To measure the effect size and to confirm that result, the researcher calculated the practical significance of the results by calculating the Eta square value (η2), which is

used to determine the degree of importance of the result whose existence was statistically proven to be (0.735), which indicates the existence of a significant and important effect of the independent variable (the training program) on the dependent variable (mathematical thinking skills) of the experimental group students.

DISCUSSION

The present study sought to investigate the effectiveness of a training program based on mathematical thinking skills on the academic achievement in mathematics among students from the upper basic stage. Through applying the quasi-experimental research design and the use of an experimental group and a control group, the researcher used a mathematical thinking skills pre and post-test.

Through the results that have been obtained, the researcher believes that the impact of the proposed training program to improve mathematical thinking skills is more effective than the traditional method in developing mathematical thinking skills among students of the upper basic stage in mathematics, which contributed to providing an educational environment dominated by group learning, cooperation and interaction among students participating in the proposed training program, as they became more active towards learning, in a more positive way in understanding new information and knowledge, and contributed to the development of their mental abilities, and the improvement of their health and psychological status, which led to an increase in the level of scientific performance of students and in turn led to an improvement in their academic achievement level.

The instillation of mathematical thinking methods led to the development of induction, deduction and logical thinking skills in order to reach correct results and mature conclusions necessary to prove the validity of mathematical issues according to the generalization of a theory or a law, or the acceptance of a case by analogy with the validity of another similar case, and the generalization that the mathematical conclusions are correct has the same nature as generalizations of experiments in practical sciences. In this type of learning, the information is not readily-delivered to the students, but the learner him/herself searches for it and accesses it through multiple sources such as scientific books, encyclopedias, and electronic sources such as the Internet, and thus the student acquires and grows knowledge, an aspect that was not available to the students of the control group.

The test was built according to studied scientific and methodological steps, which appeared in the diversity of strategies and teaching methods, in solving the problem, the challenge, and the test focus on the steps of research and investigation, and the diversity accompanying these strategies in enrichment activities and organization of ideas, had a significant impact on the development of the students' mathematical thinking skills, improving their skills, and their employment of the learned skills and enrichment questions from the test during teaching, which were applied by the trainees during the test and that was logically reflected on the thinking skills of the students participating in the training program in general and their mathematical thinking in particular, and this

also appeared through their achievement results on the test, which confirms the significant increase of the results in some skills.

The result of this study can be explained in that the teaching strategy based on mathematical thinking skills occurs in a meaningful and continuous learning, which indicates the effectiveness of retaining and keeping information for students for a longer period. The members of the experimental group learned in a way in which ideas, words and symbols are translated into mental images, which helps them to reflect on every aspect of it, by providing a collective learning environment, requiring their involvement in designing relationships between concepts, and thus installing them in their minds for a longer period of time, while members of the control group learned in a way that focused on memorizing information and facts without meditating on them.

CONCLUSION

The present study concluded that adopting training programs would significantly improve students' mathematical thinking capacity and improve their achievement in mathematics.

RECOMMENDATIONS

Based on the findings of the present study, the researcher recommends using methods, approaches and programs that develop and stimulate the mathematical thinking skills of upper basic stage students, especially in mathematics. In addition, the study recommends adopting the topic of developing mathematical thinking skills in raising the level of academic achievement as one of the objectives in the basic stage and organizing the contents of the curriculum in light of its different skills and styles. Moreover, the study recommends develop methods for evaluating teacher performance, and providing teacher preparation programs with training on methods for developing mathematical thinking skills and their role in raising the level of academic achievement among primary school students. This study has both practical and research implications. For example, one of the significant implications of this study is to design interventional training programs that aim to improve intermediate stage students' mathematical thinking and achievement. In addition, one of the research implications is to use the data collection tools adopted in this study to conduct further cross-sectional studies that aim to improve school students' mathematical thinking and achievement.

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