International Journal of Instruction *October 2023 ● Vol.16, No.4*
e-ISSN: 1308-1470 ● www.e-iji.net *p-ISSN: 1694-609X*
pp. 1025-1040

Article submission code: 20221228113505
Received: 28/12/2022
Revision: 21/05/2023
Accepted: 12/06/2023
OnlineFirst: 15/09/2023

Solving of Polyhedron Problems Based on Spatial Sense of Junior High School Students

Saffanah Pertiwi
Universitas Pendidikan Indonesia, West Java, Indonesia, *saffanahpertiwi@upi.edu*

Suhendra
Universitas Pendidikan Indonesia, West Java, Indonesia, *suhendra@upi.edu*

Nurjanah
Universitas Pendidikan Indonesia, West Java, Indonesia, *nurjanah@upi.edu*

Dadan Dasari
Universitas Pendidikan Indonesia, West Java, Indonesia, *dadan.dasari@upi.edu*

This study aims to get an overview of the Mental Action, Ways of Thinking (WoT), and Ways of Understanding (WoU) as parts of the process of solving polyhedron problems based on spatial sense. A qualitative approach was applied in this study. Data collection was carried out through several tests to measure students' spatial sense and interviews. The results show that the problem-solving process carried out by students has its uniqueness. One of the mental actions done by students when solving solid geometric problems is to imagine the figure. The formed WoT depends on the mental action taken, one of which is when students imagine the given figure to be interpreted. This then leads to the formation of WoU, which is the decision making regarding the given problem. The mental action is relatively the same in each solution. Meanwhile, the formed WoT varies in solving certain problems. Furthermore, not all students can reach the WoU stage when solving polyhedron problems; only some of them can associate the problems with the mathematical concepts that they have previously had. Inadequate mental action and WoT of the students result in the formation of meaning which leads to less than optimal WoU.

Keywords: spatial sense, mental action, ways of thinking, ways of understanding, problem solving

INTRODUCTION

Mathematics is a subject with various branches that help students to develop the ability to think logically, analytically, systematically, critically, and creatively, one which is geometry. This branch of mathematics contains the concepts of points, lines, plane figures, and solid figures which require the above ability to understand. According to

Citation: Pertiwi, S. Suhendra., Nurjanah., & Dasari, D.(2023). Solving of polyhedron problems based on spatial sense of junior high school students. *International Journal of Instruction*, *16*(4), 1025-1040. https://doi.org/10.29333/iji.2023.16456a

Nurjanah et al. (2014), geometry not only stands out in terms of deductive methods and abstract objects, but is also an effective technique for solving mathematical problems. This is supported by the statement of the National Council of Teachers of Mathematics (NCTM, 2000) that one of the standards for teaching geometry in schools is to invite students to analyse the characteristics of geometric shapes and make mathematical arguments about geometric relationships, as well as to get them to use visualization, spatial sense, and geometric modeling to solve problems. Therefore, geometry is one of the crucial materials for students to learn and master.

Considering the importance of geometry, the Ministry of Education, Culture, Research, and Technology of the Republic of Indonesia (Kemendikbud, 2014) has made geometry one of the main subjects in mathematics at junior high schools in Indonesia. This is in line with the Regulation of the Minister of Education, Culture, Research, and Technology of the Republic of Indonesia Number 64 of 2013 on Content Standards for Elementary and Secondary Education (Permendikbud, 2013), which states that one of the scopes of material in mathematics for Grades VII and VIII is geometry (including geometric transformations). One of the reasons for this consideration is that geometry is a crucial aspect for students as it includes the application of spatial sense (Nurjanah et al., 2014; Siswanto & Kusumah, 2017), which is one of the abilities needed to study solid geometry.

Spatial sense plays an important role in solving geometric problems related to visualization and space. This is in line with Nurjanah et al. (2014) who revealed that spatial sense is one of the contributing factors in studying geometry. Several previous studies have also stated that there is a positive relationship between spatial sense and mathematics learning achievement (Al-Balushi & Al-Battashi, 2013; Chen, 2022; Gilligan et al., 2017; Harmony & Theis, 2012; Tambunan, 2006; Turğut & Yilmaz, 2012). Aside from mathematics, spatial sense also has a positive role in various other fields in human life (Guzel & Sener, 2009; Rodán et al., 2019; Yilmaz, 2009). According to the results of a study by Cheng and Mix (2014), there is a significant increase in the ability to solve certain problems with only one training session on spatial sense, indicating a relationship between spatial sense and mathematical reasoning. Hence, spatial sense becomes one of the supporting factors for students to solve solid geometric problems and other problems in their everyday life.

The explanation above is contradictory to the reality at school which shows that geometry lesson is in a worrying situation as many students still experience difficulties in understanding geometry, especially solid geometry (Nurjanah et al., 2014, 2017). Several prior studies have found that students have difficulty imagining the given solid figures (Güven & Kosa, 2008; Nurjanah & Juliana, 2020; Šipuš & Čižmešija, 2012; Siswanto & Kusumah, 2017). In addition, the ways students solve problems are still very diverse.

When solving a problem using their spatial sense, students usually imagine a given object, then making a conjecture about it. This shows that students take actions in solving a problem, which is referred to as mental actions (Harel, 2008; Suryadi, 2019). According to Suryadi (2019), the preformed mental actions will later form ways of thinking (WoT). When associating WoT with previously studied concepts, a meaning is formed that leads to an understanding known as ways of understanding (WoU) (Suryadi, 2019). The uniqueness of answers given by each student is due to the diversity in a series of problem-solving steps involving mental actions, WoT, and WoU (Harel, 2008; Suryadi, 2019). In other words, mental actions, WoT, and WoU are three interrelated processes in solving polyhedron problems.

This study examines students' mental actions, ways of thinking, and ways of understanding when solving polyhedron problems based on their spatial sense. Thus, the research questions of this research are:

1. What mental actions are taken by the students when solving polyhedron problems in terms of spatial sense?
2. What kinds of ways of thinking are formed based on the mental actions performed by the students when solving polyhedron problems in terms of spatial sense?
3. Which ways of thinking lead to students' ways of understanding when solving polyhedron problems in terms of spatial sense?

Context and Review of Literature

Problem-solving

Problem-solving is the process of finding a solution to a problem (Gulo, 2008). According to Harel (2008), the process of solving mathematical problems can be seen through the triadic cycle, which consists of mental acts, ways of thinking, and ways of understanding.

Mental Actions

Harel (2008), reveals that human reasoning involves many mental actions, some mental actions that are important in learning mathematics include interpreting, guessing, concluding, proving, explaining, compiling, generalizing, applying, predicting, classifying, searching, and solving problems. Furthermore, Harel (2008), states that mental action can be learned through observing one's statements and actions because humans carry out mental actions which tend to be different depending on the field of life and professionalism. For example, a biologist, chemists, physicists, and mathematicians perform problem-solving actions, they can produce the same solution for the same problem in their respective fields but tend to differ in their nature and characteristics when carrying out these actions (Harel, 2008).

Ways of Thinking

Ways of thinking are cognitive characteristics of mental action, where these characteristics are always concluded from observing ways of understanding-cognitive products from mental action (Harel, 2008). According to Suryadi (2019), although the initial process of mental action is random, in the end, there will be a process of rearranging it so that a continuous flow of thinking is formed, this flow then forms ways of thinking. The ways of thinking that are formed will vary due to a series of mental actions that tend to be different for each individual, apart from that the diversity of abilities, experiences, and knowledge possessed by students is also a factor in the emergence of various ways of thinking construction (Suryadi, 2019).

Koichu and Harel (2007), revealed that the ways of thinking that determine ways of understanding include problem-solving approaches and evidence schemes. For example, a teacher or researcher who follows students' mathematical habits will conclude that students' interpretation of arithmetic operations is characteristically inflexible, or connected to other concepts (Harel & Koichu, 2010).

Ways of Understanding

According to Harel (2008), ways of understanding are certain cognitive products from mental actions carried out by an individual. Suryadi (2019), revealed that when the process of forming a flow of thought occurs and is in contact with a certain context so that meaning is formed, then a flow is formed which ends in understanding or can be called ways of understanding. Furthermore, Koichu and Harel (2007), revealed that ways of understanding mathematical content include the act of producing meaning or interpretation for a term, statement, or problem. Different individuals usually produce different ways of understanding, even though they have the same mental action (Harel & Koichu, 2010).

Based on the explanation above, problem-solving is a series of mental actions that cause a variety of ways of thinking, so that ways of understanding are formed which are solutions to a problem. This study examines the problem-solving process based on the triadic cycle which consists of mental acts, ways of thinking, and ways of understanding.

Spatial Sense

Spatial ability is an ability to visualize an object in space (Pujawan et al., 2020). Meanwhile, Linn and Petersen (1985, p. 1482) stated that "Spatial abilities generally refer to skills in representing, transforming, generating, and recalling symbolic, nonlinguistic information". Furthermore, Linn and Petersen (1985), divided spatial abilities into three categories, including spatial perception, mental rotation and spatial visualization. Related to this, McGee (1979), revealed that there are two main factors in spatial ability, namely spatial visualization (Vz) and spatial orientation (SO). Meanwhile, Maier (1996), states that there are five elements of spatial ability, namely spatial perception, visualization, mental rotation, spatial relation, and spatial orientation.

Based on the explanation above, what is meant by spatial ability in this study is a person's ability to visualize, rotate, and transform an object in two-dimensional space or three-dimensional space, with the following indicators:

1. Ability to distinguish between shapes
2. Ability to identify the three-dimensional object from a given net
3. Ability to identify the net of a three-dimensional object from a given figure
4. Ability to determine new states of three-dimensional objects according to defined rotation process
5. Ability to determine the relationship of an object with another object
6. Ability to recognise the identity of an object when viewed from different angles

7. Ability to consider spatial relations, where the orientation of the observer's body is an essential part of the problem
8. Ability to imagine internal movement or displacement among the parts of a configuration

METHOD

This study applied a qualitative approach based on post-positivism or interpretivism to examine the condition of natural objects, where the researcher is the key instrument. Data collection was carried out by triangulation involving tests, observations, and interviews, obtaining qualitative data for the inductive/qualitative analysis to obtain qualitative results to understand meaning and uniqueness, construct phenomena, and formulate hypotheses.

This research was conducted in October 2020 at a junior high school in Serang City, Banten Province, Indonesia. The participants in this study were 20 ninth-grade students aged between 15 to 16 years consisting of 9 boys and 11 girls who had studied syllabus-based polyhedrons in the junior high school mathematics curriculum (Kemendikbud, 2017). The instruments used in this study were tests to measure students' spatial sense and interviews to obtain additional information from the teachers and students involved about their problem-solving process. To complement these two instruments, students were also observed to get an overview of the situation when they solve the problems using observation guidelines that describe the steps taken by students in working on the problems. Furthermore, a Respondent Ability Test (RAT) was carried out through Google Meet and WhatsApp Group for 45 minutes. Meanwhile, interviews with the students were conducted through WhatsApp using open-ended questions, whereas interviews with the eighth-grade mathematics teachers of these students were done directly, also using open-ended questions.

Furthermore, the data was processed through data reduction, data presentation, and conclusion drawing (Sugiyono, 2018). Data reduction was done by grouping data that has similar characteristics, resulting in denser descriptive data and more general conclusions representing the entire data obtained. In addition, data analysis was carried out based on a theory by Harel (2008), where the process of solving mathematical problems can be seen through a triadic cycle consisting of mental action, WoT, and WoU. The data was analysed by understanding students' answers supported by the results of interviews with them regarding the steps taken when solving the problems. These steps were then divided into mental action, WoT, and WoU categories.

FINDINGS

Table 1 below illustrates students' mental actions, ways of thinking, and ways of understanding when solving polyhedron problems with spatial sense indicators.

Table 1
Stages of student problem-solving

Elements and Indicators of Spatial Sense	No	Mental action	Ways of thinking	Ways of understanding	Ways of thinking that lead to the right ways of understanding
Spatial Perception (Ability to distinguish between shapes)	1	Interpreting Imagining Applying Predicting	Look 1 and Look 2 are two different dice	Determining the die that is identical to Look 1 and Look 2	
				Combining Look 1 and Look 2	
				Applying algebraic operations to the dice in Look 1 and Look 2	
			Look 1 and Look 2 are the same die	Determining the shape of the die in other positions by rotating it (using a small cardboard shaped like a die, but without the dots on each face)	✓
				Determining the shape of the die in other positions by using the rules for placing the die based on its dots (the number of dots on opposite faces is 7).	✓
				Considering options according to the initial letter in the name	
			Look 1 and Look 2 is a repeating pattern	Considering the dots that always appear in Look 1 and Look 2, i.e. 1 dot	
Visualization (Ability to identify the three-dimensional object from a given net)	2	Interpreting Imagining Conjecturing	Determining cuboid that do not match the net	Determining unsuitable cuboid after the net is constructed	✓
				Determining unsuitable cuboid by considering the shading on the net and on the cuboid	
			Determining cuboid that match the net	Determining suitable cuboid after the net is constructed	
				Determining suitable cuboid by considering the shading on the net and on the cuboid	
Visualization (Ability to identify the net of a three-dimensional object from a	3	Interpreting Imagining Predicting	Determining the net that match the cube	Constructing the nets on the choices then comparing them to the cube	✓
				Choosing the most different answer (the one	

Elements and Indicators of Spatial Sense	No	Mental action	Ways of thinking	Ways of understanding	Ways of thinking that lead to the right ways of understanding
given figure)				where the triangles are far apart); the triangles in the other options are located close to each other	
Mental Rotation (Ability to determine new states of three-dimensional objects according to defined rotation process)	4	Interpreting Imagining	Figure 1 and Figure 2 are the same quadrilateral pyramid	Determining the pyramid by considering the identical symbols in Figure 1 and Figure 2	
				Determining the pyramid that is the inverse of Figure 1	
			Figure 1 and Figure 2 are different quadrilateral pyramids	Rotating the pyramid in Figure 2 to match the pyramid in Figure 1	✓
	10	Interpreting Imagining Applying Proving	Determining the position of the DF line segment after rotation	Drawing the prism and the DF line segment, then rotating them	✓
				Drawing the prism, the DF line segment, and the new line segment	
				Imagining the prism and the DF line segment, then rotating them	✓
				Applying the formula for geometric transformation	
Spatial Relation (Ability to determine the relationship of an object with another object)	5 and 6	Interpreting Imagining Constructing	Constructing solid figures into new, continuous solid figures	Constructing figures by considering the location of holes in some figures	
				Constructing figures by considering the factors of beauty, neatness, and balance	
				Constructing figures by considering the location of given letters and marks	✓
Spatial Orientation (Ability to recognise the identity of an object when viewed from different angles)	7	Interpreting Imagining	Observing the right face of the prism	Imagining a prism and then observing the right face of the prism	
	11	Interpreting Imagining Applying Proving	Determining the top view of a solid figure	Drawing a quadrilateral pyramid and its top view	✓
				Drawing a triangular pyramid and its top view	
				Drawing a prism and its top view	
			Determining the volume of a solid figure	Applying the formula for the volume of a pyramid	

Elements and Indicators of Spatial Sense	No	Mental action	Ways of thinking	Ways of understanding	Ways of thinking that lead to the right ways of understanding
Spatial Orientation (Ability to consider spatial relations, where the orientation of the observer's body is an essential part of the problem)	8	Interpreting Imagining Conjecturing	Determining the correct solid figure based on the known top, front, and right side views	Combining the top, front, and right-side views	
				Matching the figures in the choices with the top, front, and right side views	✓
Spatial Orientation (Ability to imagine internal movement or displacement among the parts of a configuration)	9	Interpreting Imagining	Determining what Alda sees	The figure seen by Alda is the same as the figure seen by the students	
				The figure seen by Alda is the inverse of the figure seen by the students	✓
				The figure seen by Alda is a tunnel	
				The figure seen by Alda is a cuboid	

DISCUSSION

The results of this study prove that each student has a unique way of solving the same problem. However, students who come up with the same answers do not necessarily have the same reasons for their answers. In the first stage, the mental actions that were always taken by students when solving the problems in this study were interpreting and imagining. This shows that students tend to perform similar mental actions. Furthermore, almost all students carried out several other mental actions, including conjecturing, applying, proving, and constructing. Meanwhile, a small number of students performed another mental action in the form of predicting to obtain answers to the given problems. The results of this study signify that students tend to take more than one mental action when solving a problem. This is in line with (Suryadi, 2019) who stated that the mental actions performed may not have a single type and characteristic; when solving a problem, students may take several mental actions.

In addition, this study found that when solving problems using spatial perception (the ability to distinguish between different shapes), a small number of students tried to recall the material they have studied previously regarding the placement of dice. This is in line with the findings of several prior studies that there will be moments when students try to recall previous materials when solving a problem (Marzano & Kendall, 2007; Prayekti et al., 2020). Marzano and Kendall (2007) further revealed that when given a question or problem, students will enter the first stage, namely the retrieval or recall included in the cognitive system in the taxonomy. In Marzano and Kendall's taxonomy, retrieval begins with recognizing, recalling, and executing. This means that in dealing with a problem, students will try to interpret the meaning of the problem then relate it to the material they have learned before finally solving the problem. Meanwhile, other studies have found that students' skills, knowledge, and experience play an important role in their problem solving ability (Nurhasanah, 2019; Suryadi, 2019). This will eventually have a role in the formation of students' ways of thinking.

Ways of thinking, which is the next stage of problem solving, are formed depending on the mental actions taken when solving the problems. For example, one of the mental actions performed by the students when solving a problem with spatial perception was interpreting. However, despite taking the same mental action, students' interpretations of the problem were not necessarily the same as one another. Likewise, when students performed the mental action of proving to solve the spatial orientation problem by drawing the figures, they did not necessarily draw the same figures. From these findings, this study revealed that the students took more than one mental action when solving a problem. In this regard, Suryadi (2019) stated that a series of different mental actions will lead to a variety of ways of thinking. This means that the mental actions performed by students when solving a problem greatly affect the formation of their ways of thinking.

Regarding problem solving with spatial perception, students carried out four mental actions, i.e., interpreting, imagining, applying, and predicting, resulting in the formation of three ways of thinking. In different cases, although the students also took three mental actions (interpreting, imagining, and constructing) in solving problems with spatial relation, only one way of thinking was formed, namely constructing solid figures into new, continuous solid figures. This shows that the mental actions performed by students can form one or more ways of thinking, depending on the mental action itself. This finding is in accordance with the statement of Suryadi (2019) that although the initial process of mental action is random, a process of rearrangement will eventually occur, forming a continuous flow of thought which then forms ways of thinking. Thus, if the students carry out a single mental action, the way of thinking formed is not necessarily single, and vice versa.

The following is one of the questions on the Respondent Ability Test (RAT) to distinguish ability in different forms indicators. In this case, students are expected to be able to distinguish identical shapes from known shapes.

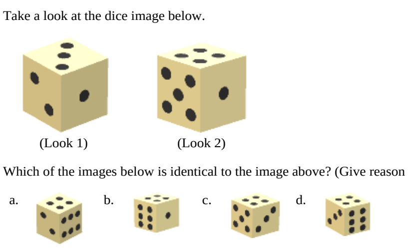

Figure 1
Question about the ability to distinguish between different shapes

It was found that when students solved the above problem (Figure 1) based on the spatial perception indicator, three ways of thinking were formed, namely: 1) Look 1 and Look 2 are two different dice; 2) Look 1 and Look 2 are the same die; and 3) Look 1 and Look 2 is a repeating pattern. When students solve problems number 2, 4, and 11 with the abilities to identify the three-dimensional object from a given net, determine new states of three-dimensional objects according to a defined rotation process, and recognise the identity of an object when viewed from different angels, two ways of thinking were formed. This indicates that students tend to have different ways of thinking when solving the same problem. This is in line with a study by Harel and Stevens (2011) which found that a problem can be solved in various ways. This finding also shows the diversity of ways of thinking, even though the mental actions taken tend to be the same. This is because the same mental action does not necessarily have the same nature and characteristics, which may lead to the possibility of the formation of diverse ways of thinking. This supports the statement of Harel (2008) that although scientists in different fields produce the same actions and solutions, the nature and characteristics when performing these actions tend to be different. In this regard, Suryadi (2010) argued that the process is under the control of the individual doing the action. Thus, even though the students came out with the same answers, the process they went through was not necessarily the same.

When the students solved visualization (number 3), spatial relation (number 5 and number 6), spatial orientation (number 7 to number 9), and mental rotation (number 10) problems, only one way of thinking was formed. This proves that in certain problems, there is no diversity of ways of thinking due to inadequate mental actions performed by students, thus not facilitating the formation of ways of thinking.

This study also found that not all students who tried to recall the previous materials could form the expected ways of thinking. For example, when solving problems with the ability to identify the three-dimensional object from a given net, all students tries to recall the material related to the nets of solid figures, but not all of them were able to construct the nets given properly. This is due to inadequate skills and knowledge of the students which make them unable to form the expected ways of thinking. This is aligned with the results of a previous study conducted by Ayuningrum and Setiawan (2018) that students without the necessary skills will have difficulty solving non-routine problems. Furthermore, a small number of students even chose answers randomly without even knowing the concepts, such as by considering the initial letters of their names. A small number of students also answered essay questions by copying their friends' answers, indicating that they do not care about the problem-solving process and only focus on the final answers to get points. Nurhasanah (2019) also found a similar result in her study that students only focus on the final solution but do not understand the meaning of the concepts and processes carried out to obtain the solution. Students who do not understand the given problem from the beginning cannot solve the problem appropriately.

The ways of understanding formed in the last stage of the problem-solving process depend on the mental actions taken and the ways of thinking formed. However, not all ways of thinking formed can lead to the right ways of understanding. Based on the data displayed in Table 1, it is known that some students could solve problems by applying the materials they have learned previously. Meanwhile, students with limited knowledge took inadequate mental actions, thereby forming inadequate ways of thinking. Thus, not all ways of thinking were related to the context and led to expected ways of understanding.

As stated previously, only one of the three ways of thinking formed in the spatial perception problem produced meaning that led to the formation of ways of understanding. Likewise, of the two ways of thinking formed in solving problems number 2, 4, and 11, only one way of thinking produced meaning that triggered the formation of ways of understanding. This is due to the inadequate ways of thinking formed, so that the formation of a meaning to reach ways of understanding is not well facilitated. This study found that one way of thinking leads to ways of understanding in the spatial perception problem. This is because students can relate the problem to concept they have learned before regarding dice placement. This is in accordance with the opinion of Suryadi (2019) that the process of forming a flow of thought related to a certain context to form a meaning will eventually form ways of understanding. Therefore, to reach ways of understanding, a connection between the problem and previously learned concepts is highly needed.

Only a small number of students in this study could relate the problem to previously learned concepts. For example, for one of the mental rotation problems, one student was able to mention the correct degree of rotation. However, this student has not been able to imagine the result of the rotation properly, thus experiencing an error. A study by Azustiani et al. (2017) found a similar result that most students have difficulty imagining the rotation that occurs in a geometric object. In addition, the error experienced by this student is due to inadequate ways of thinking which was unable to facilitate the formation of ways of understanding.

In the spatial perception problem, only a small number of students could relate the problem to the concept of dice placement. Among these students, one of them was able to solve the problem with the help of a visual aid in the form of used cardboard. This student formed the expected ways of thinking and could get the meaning that led to the formation of ways of understanding. This signifies that teaching aids can help students understand abstract mathematical problems, especially in geometry. Several previous studies have also found that the use of media or props can help students in learning geometry (Elvi & Nurjanah, 2017; Nurjanah et al., 2020) and can even improve students' mathematics abilities (Rohendi et al., 2023). Meanwhile, a small number of students experienced errors in imagining the shape of the dice in the problem, indicating that they have difficulty imagining a geometric object. This is aligned with the results of several prior studies that many students still experience difficulties in understanding solid geometry (Nurjanah et al., 2014, 2017; Siswanto & Kusumah, 2017). These difficulties can be overcome by using suitable media, props, or visual aids.

In one of the spatial orientation problems, none of the students were able to form a meaning that may lead to ways of understanding. This is because students tend to see a figure from the front (the figure they saw in the question), instead of orienting themselves in the position requested, resulting in inaccurate answers. This is in line with the results of a study done by Rohmatunnisa (2019) which revealed that students have difficulty imagining the invisible parts of a given geometric object. Thus, students' point of view becomes one of the important factors in solving solid geometric problems.

In addition, this study found that a small number of students did not recognise the polyhedron requested in one of the spatial orientation problems, so they drew different polyhedrons. One of the students even solved the problem by applying the formula for the volume of a pyramid. This may happen because the learning process is inadequate, and the given material on geometry focuses more on the use of formulas for surface area and volume of polyhedrons. These findings are in line with the statement of Syahputra (2012) that mathematics teachers generally place more emphasis on aspects of geometry such as formulas. Meanwhile, an understanding of the problems given is more needed in solving geometric problems as they are not only related to formulas.

A small number of students was found to choose answers without clear reasons; some of them even merely copied their friends' answers without knowing the process of solving the problems. This indicates inadequate skills, knowledge, experience, and mentality of the students that some of them cannot determine the right answer or the reason. A small number of these students still cannot relate the problems to the concepts they have studied before and even have forgotten the previous materials. This is in line with the results of a study by Nurhasanah (2019) which found that students are unable to relate previously learned concepts. As a result, students cannot solve other problems related to these concepts.

CONCLUSION

Based on the analysis, it can be concluded that:

1. Each student tends to take mental actions that are not much different from one another and all students carry out several mental actions when solving a problem. The mental actions performed by the students include interpreting, imagining, predicting, conjecturing, applying, proving, and constructing the given geometry. The mental actions taken by the students depend on their knowledge and skills.

2. The ways of thinking tend to vary in some problems, but not in several other problems. This depends on the mental actions performed by the students. Students with the same mental action do not necessarily have the same ways of thinking. Furthermore, ways of thinking are also formed based on students' understanding of the concepts studied previously. Only a small number of students have a problem-solving approach that leads to the right solution, whereas another small number of students only focus on the final answer without thinking about the process.

3. The formation of ways of understanding depends on the ways of thinking and mental actions of the students. Not all ways of thinking of the students lead to meaning so as to reach ways of understanding. This is because not all students can apply the mathematical concepts they have learned when solving the problems. In addition,

inadequate mental action and ways of thinking result in a lack of ability to develop ways of understanding.

ACKNOWLEDGMENTS

The main author expresses the deepest gratitude to her thesis supervisor, Prof. Dr. Didi Suryadi, M.Ed., for his support and guidance. In addition, the author would like to profusely thank Lembaga Pengelola Dana Pendidikan (LPDP/Indonesia Endowment Fund for Education), which is part of the Ministry of Finance of the Republic of Indonesia, for funding her master's education and supporting this publication.

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