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# The Effects of Using Video-Based Instruction in Solving Fraction Computations of Students with Autism Spectrum Disorder

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The aim of this research was to assess whether using video modeling (VM), concrete manipulatives, a self-monitoring checklist, and practice for comprehension check impacted the extent to which five primary school students with Autism Spectrum Disorder (ASD) could accurately solve problems involving fractions. The sample of the study was purely comprised of five students owing to the challenges associated with eliciting approval to extend the study to a larger sample. A single-case multiple probe between participants experimental approach in its quasi-experimental design was employed to evaluate whether a significant association could be identified between these variables. The results revealed that from baseline to intervention, all five students solved simple proper fraction problems with greater accuracy and four were able to apply this ability to solve problems involving whole proper fractions. Given these results, the researchers recommend that to cater for the disparate learning needs of students with ASD across a range of settings, teachers should consider implementing interventions comprising VM and concrete manipulatives in conjunction with certain behavioural techniques. Owing to the challenges associated with eliciting approval to extend the study to a larger sample.

Keywords: primary school, autism, mathematics, video-based instruction, concrete manipulatives, self-monitoring strategy

## INTRODUCTION

To enable autistic students to realise their future potential in areas such as employment and independent living, they should be provided with appropriate academic instruction whilst at school (Wong et al., 2021). Almost all autistic students can live successful lives if they are taught mathematics skills and basic academic concepts(Stroizer et al., 2015). With respect to the former, all students must acquire both basic and more sophisticated conceptual and computational skills (National Council for Teachers of Mathematics (NCTM, 2000). Although those with autism are not defined by academic difficulties, a substantial number do find it challenging to learn mathematics (Wei et al., 2015). For instance, challenges with executive functioning that may impact organization, problem solving, and self-management skills (Ozonof & Schetter, 2007), decreased on-task behavior and engagement (National Research Council, 2001), and perceived challenges

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in processing abstract concepts in mathematics (Rourke & Strang,1978) may contribute to difculties in learning mathematics concepts among autistic students. They experience these difficulties to a greater extent than those without disabilities, a notable example being the problems they encounter when learning fractions (Hecht & Vagi, 2010; Misquitta, 2011). Fractions tend to be especially difficult for students with disabilities in mathematics (NMAP, 2008; Sanders et al., 2005). One reason fractions may be difficult is that students may apply rules of whole numbers erroneously to rational numbers (Ni and Zhou, 2005). For example, multiplying two positive rational numbers results in a larger product; however, multiplying a number by a fraction may yield a smaller product. Proficiency in fractions requires that students attend to two quantities (i.e., numerators and denominators) simultaneously and recognize their relationship (NCTM, 2013).

Although difficult, fractions are a vital mathematical skill for the majority of students (Hecht & Vagi, 2010; Mazzocco et al., 2013). However, students have reported an improvement in the acquisition of mathematical skills when extremely clear instruction with visual and concrete examples is provided to enhance their understanding of a range of mathematical concepts (Bouck et al., 2018; Satsangi et al., 2019; Yakubova et al., 2016). A type of visually supported instruction that uses technology is a video-based instruction, an evidence-based practice for teaching a wide range of skills to autistic individuals from early childhood to young adulthood (Steinbrenner et al., 2020). Video-based instruction can be used to provide systematic instruction with consistent vocabulary and explicit modeling for the individual to watch and imitate the target skills or concepts (Hughes & Yakubova, 2019). VM is established as an evidence-based practice to teach a wide array of skills to students with ASD from elementary to high school grades according to The National Professional Development Center review of evidence-based practices (Wong et al., 2015). Therefore, the aim of this research was to assess whether the accuracy with which primary school students with ASD can solve problems involving proper fractions is impacted by the provision of a practice for comprehension check, a self-monitoring checklist, and a VM instructional package with concrete manipulatives.

**Statement of the problem**

A robust understanding of mathematics needs to extend beyond whole numbers to encompass fractions and decimals (Fennell, 2007). Moreover, knowing how to deal with fractions is purported to be the most essential fundamental ability (NMAP, 2008) and can potentially predict future accomplishments in mathematics (Bailey et al., 2012; Watt & Therrien, 2016) up to five years hence (Siegler et al., 2010). It has also been argued that inadequate knowledge of fractions means students are not as well prepared for other mathematics classes as they need to be (Sanders et al., 2005). Students with disabilities in the domain of mathematics often find fractions particularly challenging (NMAP, 2008; Sanders et al., 2005). ASD is one such disability, and is characterised by (a) repetitive behaviours and limited activities and interests, and (b) deficiencies in social communication and interaction (American Psychiatric Association (APA), 2013). These features make it difficult for those with ASD to successfully develop social relationships, perform well academically, become independent, and have an acceptable quality of life (Schall et al., 2012). For instance, Clarke et al. (2021) investigated vocational activity trajectories in young adults with autism and found that continuous efforts to support these individuals were required for them as they worked toward the achievement of independence. Wehman et al. (2014) identified one of the critical elements in autism-specific interventions as the use of self-monitoring. It has been viewed by relevant stakeholders as an important skill for individuals with autism to develop (Hume et al., 2009; McDonald & Machalicek, 2013), because such features extend throughout the course of their lives and pervade all elements of their everyday existence (Hendricks and Wehman, 2009). Despite this, few studies have been conducted on academic instruction for people with ASD and those that have been conducted typically focus on literacy (Pennington, 2010; Spencer et al., 2014). Thus, although the number of occupations requiring sophisticated knowledge and skills in mathematics is increasing (Bureau of Labor Statistics, 2014), there is a dearth of research on the way in which mathematics is taught to students with ASD (Spencer et al. 2014). Yet a large number of such students are classified as having a disability in mathematics (Mayes and Calhoun, 2006) or find it challenging (Meyer and Minshew, 2002; Rourke and Strang, 1978), and at least a quarter of these find mathematics to be more difficult than learning vocabulary (Williams et al., 2008).

*Research questions*

The primary research questions were as follows:

(1) Does the accuracy with which students with ASD solve problems involving proper fractions improve following the implementation of an intervention package comprising VM instruction, concrete manipulatives, practice for comprehension check, and a self-monitoring checklist?

(2) Do such students with ASD extend and apply their ability to solve problems involving proper fractions to problems involving improper fractions?

**Theoretical Framework**

**Fractions Interventions for Students with ASD**

Because mastering fractions is now viewed as essential for achieving mathematical success and proficiency, there is a growing emphasis in curricula on teaching fractions (Booth and Newton, 2012; NMAP 2008; Vukovic et al., 2014). Due to their ubiquitousness in people's everyday lives, knowing how to deal with fractions is argued to support independence in adulthood (Jordan et al., 2017). However, Maccini and Gagnon (2000) found that a substantial number of special education teachers knew little about the standards for national mathematics and tended to concentrate primarily on basic mathematics. The standards currently focused on by the few instructional models available for teachers to use with students with ASD concentrate on basic numerical operations or the management of money (Browder et al. 2012; Spencer et al. 2014).

Four categories of interventions were identified in a review by Misquitta (2011) of studies focusing on fraction-related support for students having difficulty with mathematics. These were anchored instruction (i.e., videodiscs addressing real-life problem-solving scenarios), the concrete–representational–abstract (CRA) framework, strategy instruction (e.g., mnemonics), and direct instruction. Misquitta (2011) found that through explicit teaching, such approaches resulted in better performance by students in work on fractions. This is perhaps unsurprising given that explicit instruction, the pedagogical basis of the CRA framework, is based on evidence and recommended by practitioners for use with students presenting with both-incidence and low-incidence disabilities (Browder et al., 2012; Doabler & Fien, 2013; Gersten et al., 2009; Root et al., 2017). The benefits of both direct and strategic instruction for students with disabilities in mathematics have also been identified in earlier meta-analyses (Swanson & Hoskyn, 1998). In a recent study, VM in conjunction with a self-regulation strategy was utilised by Hughes (2019) to successfully instruct a middle school student with ASD on how to go about simplifying fractions. Moreover, the student was able to sustain such skills over time. Bouck et al. (2019) reported that making use of virtual manipulatives through a virtual-representational-abstract instructional sequence enabled a middle school student with ASD to instantly develop the ability to identify problems involving fractions.

With decades of evidential support for its application, the CRA framework has been applied across numerous domains of mathematics (e.g., subtraction, multiplication, fractions, and algebra), (e.g., Agrawal & Morin, 2016; Bouck et al., 2017; Butler et al., 2003; Flores, 2010; Jordan et al., 1998; Miller & Mercer, 1993; Underhill, 1977). Through explicit instruction, the CRA framework involves the educator demonstrating and thinking aloud to make the mathematics extremely clear to the student, directing the student as they strive to solve the mathematical problem, and then allowing them to solve the problem themselves. Using CRA, students are instructed to solve mathematical problems in a systematic fashion, using concrete manipulatives followed by representations or drawings, and then abstractions in the form of mathematics (Agrawal & Morin, 2016). In all three phases of the CRA framework: concrete manipulatives, representational, and abstract, explicit instruction is entrenched. Even though few studies have been conducted on the use of CRA framework in teaching fractions, it is generally viewed as a research-informed and evidence-based intervention for students with developmental and learning disabilities (Agrawal & Morin, 2016; Bouck et al., 2017; Flores et al., 2014).

Despite the paucity of studies, two largely identical frameworks for teaching fractions to students were compared by Butler et al. (2003) in a group study of middle school students with disabilities. In this research, the CRA framework was compared to a framework that involved just two of the phases: representational and abstract. Comparing the posttest results with those of the pretest, Butler et al. found that although an improvement was evident in both groups, this was greater for students using the CRA framework for all the subtests that were applied. However, this was only significant for the subtest that evaluated whether students were able to assess a fraction of a given amount (e.g., circle 3 or 4 of the 24 dots).

The current researchers were able to identify three empirical studies focused on fractions that were directly relevant to students with ASD. In the first of these, Yakubova et al. (2015) assessed the use of an intervention using VM in conjunction with a self-monitoring checklist to teach high school students with ASD how to solve word problems by subtracting mixed fractions with rare denominators. The results indicated that the intervention was successful, with a mean change of 94% for all students and a follow-up assessment revealing that such skills were maintained. In the second study, Hughes (2019) successfully used VM in conjunction with a self-regulation strategy to instruct a middle school student with ASD on how to simplify fractions. Moreover, such skills were sustained over time. In the third study, Bouck et al. (2019) successfully used virtual manipulatives presented through a virtual-representational-abstract instructional sequence to enable a middle school student with ASD to instantly develop the ability to identify problems involving fractions.

*Video modeling instruction with concrete manipulatives*

A review of evidence-based practice by The National Professional Development Center concluded that VM is an evidence-based intervention that can be employed to teach a diverse range of skills to students with ASD from elementary grades through to high school (Wong et al., 2015). Similar results were found in a systematic review by Hughes and Yakubova (2019). In essence, VM entails playing a video recording of methodical and explicit instruction to a student before they attempt to complete a designated task in mathematics (Hughes & Yakubova, 2019). Several studies using VM have revealed that is has a positive impact on learning, especially when applied in conjunction with academic and behavioural techniques such as manipulatives, the concrete-representational-abstract [CRA] framework, and self-monitoring checklists (e.g., Hughes, 2019; Hughes & Yakubova, 2019).

There is a substantial amount of evidential support to suggest that concrete or virtual mathematics manipulatives enable students with disabilities to understand abstract concepts at a much deeper level (Bouck & Park, 2018; Marley & Carbonneau, 2014). For instance, such students (across all grades) exhibited a greater capacity to solve computation and word problems when VM instruction was applied in tandem with self-monitoring checklists and concrete manipulatives (Hughes & Yakubova, 2019). Past studies using concrete manipulatives have concentrated on comparing them to virtual manipulatives with or without VM or on their use within the CRA instructional framework (Bouck et al., 2014; Stroizer et al., 2015; Yakubova et al., 2016). All reported that when used in conjunction with concrete manipulatives and self-monitoring checklists, VM was an effective approach to apply with such students.

Manipulatives can be effectively utilised both within and outside a CRA framework to teach mathematical concepts to students across all grade levels (Bouck & Park, 2018; Peltier et al., 2019). They are classified as a best practice (National Mathematics Advisory Panel [NMAP], 2008) and their use has been strongly supported by The National Council of Teachers of Mathematics [NCTM] (2000). Most studies in this field have concentrated on the smooth advance from concrete (or virtual concrete) to abstract practice, with the gap between the two bridged by the supported use of semi-concrete or pictorial representations. Bouck and Park (2018) conducted a review of studies published from1975 to 2017 which employed manipulatives to teach mathematics to students with disabilities. They reported that out of the 36 studies reviewed, just seven evaluated the impact of manipulatives outside the CRA framework and only a tiny number involved students with ASD. Therefore, to assess the independent effects of manipulatives outside the CRA framework as a stand-alone tool, additional research must be conducted.

## METHOD

### Setting

The participants were five primary school students with ASD who attended an independent day school for children with disabilities in the city of Dammam. The inclusion criteria were as follows: (a) in receipt of a primary diagnosis of ASD, (b) additional support needed for mathematics in accordance with the recommendations of their teacher, (c) had not previously used VM in mathematics, (d) teachers reported no vision, hearing, or gross motor difficulties that would limit their capacity to utilise VM instruction, and (e) were willing to take part in the research.

Creswell (2003) asserts that it is the responsibility of the researcher to make sure participant's rights are upheld. Accordingly, the aim of the study was explained to the parents/guardians of each participant in as much detail as possible. The parents/legal guardians then signed an informed consent permitting their child to take part while the participants freely signed assent forms before commencing the study. All those involved received a copy of each consent form. The researcher also maintained the anonymity of participants by not disclosing their identities or any other identifying information when reporting any aspect of the research.

The setting for the research was a government primary school for children with intellectual disabilities, ASD, SLD, and other forms of impairment from first through to sixth grade. All students were provided with funding from the government. To ensure the individual needs of students were met, there was a staff to student ratio of 1:5, which was reduced to 1:1 when a coordinated team approach was applied. The research activities took place in the mathematics laboratory twice a week during a period in which students were provided with support for their mathematics skills. Within the mathematics laboratory room were 20 internet-connected computers for students, a computer for the teacher, a colour printer, interactive whiteboards, and a projection system for presenting sample materials and programmes. Those present in the mathematics laboratory during the research were the participants, the interventionist (the researcher), and the person collecting reliability data.

Having earned master's and doctoral degrees in both mathematical disabilities and technology from the UK, the researcher had substantial research and practical experience working with elementary school children to young adults with ASD. Before commencing the research, the researcher was trained in how to carry out the intervention and the associated research activities.

### Participants

The first participant was a fifth grade male aged 10 who had been diagnosed with autism at 3 years old by a developmental pediatrician. Unfortunately, the doctor's report did not contain any specific information on the tests performed to make this diagnosis.

Therefore, to assess whether he was eligible according to IDEA, an additional, in-depth assessment was carried out by a school psychologist who also diagnosed him with autism. According to his IE, school records also reveal secondary diagnoses of SLD in reading, writing, and mathematics, and an auditory processing disorder. The most up-to-date scores on the MAP-M indicate that his mathematics skills were in the third percentile as peers of the same age. His most recent WJ-IV reveals scores ranging from 56 to 63 on the mathematics component, which compared to peers of the same age, positions him in the very low achievement range. Although shy, he was keen to engage with the interventionist. Depending on his mood during sessions, his focus and on-task behaviour often varied.

The second participant was a sixth grade male aged 11. According to his IEP, he has been diagnosed with ASD, and since preschool had been receiving support with speech due to a language delay and special education services for an identified emotional and behavioral disorder. He has also received a diagnosis of 'ADHD mixed R/O PDD' (Rule-out Pervasive Developmental Disorder). Having been diagnosed with high functioning autism by a licensed psychologist in 2019, he started receiving services for autism as opposed to emotional and behavioural disorder. In his most recent WJ-IV, his scores ranged from 55 to 62 in the mathematics component, which compared to peers of the same age, positions him in the very low achievement range. Assessments and observations conducted recently indicate that he would be able to solve mathematics problem at a second-grade level and fractions at a low fourth-grade level. The interventionist found him to be a shy boy who, when engaging in tasks, would quickly become frustrated.

The third participant was a fifth grade male aged 10. According to his IEP, school records reveal that he has received a primary diagnosis of ASD and secondary diagnoses of epilepsy, attention deficit hyperactivity disorder (ADHD), and specific learning disability (SLD)in. The Measure of Academic Progress—Math (MAP-M) obtained most recently reveals scores that, in comparison to peers of the same age, means he is positioned in the 1st norms percentile and in the low-average range for a third-grade student. His current IEP goals suggest that when working with guided instruction on third- to fourth-grade level mathematics problems, he performs with 70% accuracy, but this falls to 45% accuracy without such instruction. The interventionist found that when engaging in mathematics tasks, he quickly became frustrated and displayed the unusual and slightly alarming habit of placing the wooden figures in his mouth.

The fourth participant was a sixth grade male aged 11. At the age of 20 months he was given a diagnosis of ASD by a paediatric psychologist. Unfortunately, the doctor's report did not contain any specific information on the specific tests that were performed to make this diagnosis. Therefore, to assess whether he was eligible according to IDEA, an additional, in-depth assessment was carried out by a school psychologist who also diagnosed him with autism. According to his IEP, school records indicate that he has also received secondary diagnoses of SLD in reading, writing, and mathematics, and an auditory processing disorder. The scores he recently obtained on the MAP-M indicate that in comparison to peers of the same age, his skills in mathematics are in the fifth percentile. In addition, his most recent WJ-IV revealed scores ranging from 54 to 62 on the mathematics component, which compared to peers of the same age, positions him in the very low achievement range. The interventionist observed that he found it difficult to pay attention and follow instructions during lessons.

The fifth participant was a sixth grade male aged 11. According to his IEP, school records indicate he has been given a primary diagnosis of ASD and secondary diagnoses of epilepsy, specific learning disability (SLD), attention deficit hyperactivity disorder (ADHD), and language development delays. Scores he recently obtained on the Measure of Academic Progress—Math (MAP-M) reveal that in comparison to peers of the same age, he is positioned in the 1st norms percentile, and is in the low-average range for a third-grade student. Furthermore, on the WJ-IV completed most recently, he obtained scores ranging from 54 to 61 on the mathematics component, positioning him in the very low achievement range. Finally, recent assessments and observations suggest he would be able to solve mathematics problems at a second-grade level and those involving fractions at the level of a low fourth-grade student. The interventionist observed that the participant was keen to work with him.

**Independent variable**

The intervention was the independent variable and comprised four methods: VM recorded from a point-of-view perspective, concrete manipulatives, a comprehension check, and a self-monitoring checklist. During each intervention session, participants viewed the VM clip, undertook a comprehension check with the interventionist based on a sample problem, and then solved probe questions using a self-monitoring checklist and manipulatives. If they felt it necessary, participants were permitted to view the VM clip again.

**Video modeling instructional clip**

For the target task using the concrete manipulatives, the researcher and two members of staff who specialised in programming created a bespoke VM clip. Displaying only the hands of the model and the task they were explaining, it was recorded from the point-of-view of the first-person, and was therefore designated a point-of-view video modeling (POVM) clip. To achieve this, the task was video recorded using the document camera IPEVO (https://www.ipevo.com/) as this had the capacity to zoom in on the task/worksheet without needing anyone else to hold the camera. Using the same format as the worksheets supplied to participants during baseline and intervention sessions, the VM clip modelled how to solve a sample problem involving addition of fractions and another involving subtraction of fractions. The method used was broken down into individual steps which were presented individually using a printed checklist that matched a self-monitoring checklist, and also verbally. The VM clip thereby offered a form of instruction that was both methodical and explicit. In terms of the specific steps, the instructor in the video began by reviewing numerators and denominators, and then, using the fraction tile manipulatives, demonstrated how to build a fraction. The first step on the problem-solving checklist was then displayed along with the first sample question (addition). Referring to the first step, the instructor read the question out loud. The problem was then solved by the instructor, who presented and read out loud each step in the problem-solving process while modelling it using suitable materials such as pencils, fraction tiles, and so on. The exact same process was then repeated for the second example (subtraction) and the overall clip lasted for 4 min. The video clip was then uploaded on to the desktop for participants to use.

**Concrete manipulatives**

To assist participants in understanding the problem at a conceptual level and provide a practical demonstration of the task, concrete manipulatives in the form of fraction tiles were used. These were colour-coded by denominator and displayed fractions ranging from 1/2 to 1/12, including a whole fraction tile to represent the number 1. In the VM clip, participants were instructed to solve the fraction problems by visually creating the fractions using the tiles.

**Self-monitoring checklist**

Using the colour coding portrayed in the video clip, the self-monitoring checklist presented the sequential tasks used to solve the fraction problem in the VM clip. Participants could then use this as a paper prompt for each of the steps employed in solving the problem. The purpose of doing this was so that participants would not need to rely on prompts from adults to jog their memory. The self-monitoring procedure was effective for students and resulted in immediate increases in student independence comprehend fractions and solve simple problems involving proper fractions. In practice, the list was only referred to by the fifth participant, indicating that all the others could recall the steps. Furthermore, after using it for the first two sessions, the fifth participant became much less reliant on it in later sessions.

**Comprehension check**

Having viewed the VM clip in full, the interventionist then engaged in a short practice session with the participants to ensure they had understood everything. This also gave them the opportunity to offer personal support so that participants were able to acquire the requisite skills quickly. While presenting the comprehension check, a student-centred approach was employed by the interventionist whereby they presented the practice problem in a format similar to that used in the VM clip and in study sessions and asked the participants to demonstrate how they would go about solving the problem. To guide them, the participant could use the VM, fraction tiles, and self-monitoring checklist, providing a visual demonstration with the tiles and then writing down the answer, or verbally explaining how they would tackle the problem. If they found a particular step in the process difficult, they would be asked by the interventionist to refer to the self-monitoring checklist. If they continued to find the process difficult, questions and prompts would then be used to guide them in creating the fractions using the fraction tiles, correcting themselves where necessary. Example questions were: ‘For the fraction 5/8, which fraction tiles do we need?’ and ‘Are you adding or subtracting?’.

**Dependent variable and measurement**

The dependent variable was the percentage accuracy with which addition and subtraction problems using proper fractions and appropriate denominators were solved. Because participants were prompted to use the fraction tiles, the interventionist only used denominators that matched the fraction tiles (1/2 to 1/12) and numerators ranging from 1 to one fewer than the number in the denominator (e.g., 3/4 or 5/6). In the session, participants were presented with five proper fraction problems explained in a suitable context; for example, ‘5/12 cup of flour take away 4/12 cup of flour’, which was also displayed underneath in the format of an equation; for example, ‘5/12 – 4/12 =?’. The use of a dual format enabled participants to link this skill to concrete examples of tasks they may undertake every day. Participants then had to write down the correct solution to each problem. Before commencing the baseline phase, researchers created a list of appropriate problems and then chose five to use for each session. The problems used during each session were different. The researchers chose the target topic according to the mathematics goals stated on each participant’s IEP and our initial assessment of their ability to compare and calculate fraction problems. For each generalisation phase, the problems used were presented in a format identical to that employed in baseline and intervention sessions but contained a whole proper fraction (e.g., 1 ¼ cup of cream plus ¼ cup of milk, 1 ¼ + ¼ = ?) and used fraction values with the denominators 1/3 to 1/12, and numerators ranging from 1 to one fewer than the number in the denominator. Instead of improper fractions, all fractions were presented as mixed fractions less than 2. For the generalisation probes, a single fraction tile denoting 1 could be utilised by participants along with the other fraction tiles. To collect data on the percentage accuracy with which fraction problems were solved, permanent product recording was used by the researchers (Ledford et al., 2018). When each session was complete, the percentage accuracy of responses to all five questions were determined by the interventionist. The mastery criterion was 100% accuracy for two consecutive sessions.

**Experimental design**

To determine the effects of the intervention on the accuracy with which five students with ASD were able to solve fraction problems, a single-case research design involving a multiple probe across five students was employed (SCRD; Gast et al., 2018). This design was selected to: (a) enable the researchers to assess whether there was a causal relationship between the independent and dependent variables and (b) to continually collect baseline data as part of a multiple baseline design that might have exerted non-desirable effects on participants, such as the second participant, who may have become frustrated or exhibited challenging behaviors. This made it possible to replicate the effect across all participants with at least three attempts at achieving the effect at three different time points (Kratochwill et al., 2013).

**Ethical procedures**

Before and during each study session, participants were informed that they were taking part on a voluntary basis, which meant they could abandon the session and indeed the study at any time without any adverse consequences. They were also informed that there would be no negative consequences for failing to solve a problem or answering it incorrectly. Participants were also offered verbal reinforcement following each session; for example, ‘that’s brilliant, thank you for your wonderful work’.

**Baseline**

In accordance with the multiple probe design, the baseline phase comprised at least five sessions for each participant, conducted at different points over time. Participants progressed from the baseline to intervention phase when a visual inspection of their data revealed an increase in accuracy over the course of a given session. However, all participants completed a minimum of five baseline sessions, and then continued until a stable or downward trend was evident, following which the intervention was introduced. Participants were informed by the interventionist that the session was about to commence and that they would be presented with worksheets containing five fraction problems which they needed to solve. They were asked to do their best to solve these. No instruction or support was provided to participants during the baseline phase.

**Intervention**

The intervention phase lasted for at least five sessions and continued until the mastery criterion of 100% accurate responses for two consecutive sessions was achieved. Following the What Works Clearinghouse (2017) design standards, at least five sessions of data were collected by the researcher, even if the mastery criterion was reached by the participant in the first couple of sessions. This made sure that any rise in the attainment of skills by participants was not attributable to chance. Prior to the commencement of each session, the interventionist explained to participants what it was they were being asked to so, which was to view the video clip, use manipulatives to solve a practice problem to confirm their understanding of the task, and then use the self-monitoring checklist and manipulatives to solve five problems presented on a worksheet. Each participant then watched the VM clip individually and, based on what the instructor had told them, used manipulatives to solve a practice problem in order to assess their level of understanding. The second and third participants often took charge in the practice sessions, solving the practice problems on their own with or without the use of manipulatives. By contrast, to correctly solve the problem, the remaining participants sometimes had to be prompted to refer to their self-monitoring checklist on how to match denominators or use the manipulatives. Each participant was then presented with the worksheet containing five problems and directed to solve them using the manipulatives and self-monitoring checklist. If the interventionist was asked to provide assistance to help solve the problems, they referred the participant to the self-monitoring checklist or reminded them that they could re-watch the video clip if they felt this would help. During the first two intervention sessions, the first, fourth and fifth participants were advised to use the self-monitoring checklist and fraction tiles to help them solve the problems. As sessions progressed, these participants reduced their use of these tools and were able to solve the problems at a more abstract level. By contrast, the second and third participants did not use the self-monitoring checklist or re-watch the video clip when solving problems. They understood straight away what was required having watched the VM clip.

**Generalisation**

The purpose of this phase was to determine the degree to which participants could apply the steps taken to solve simple proper fractions to then solve whole proper fractions. All participants attended three sessions in the generalisation phase, apart from the second participant who was only able to complete one session as the school year had ended. The procedures were largely the same as those employed in the baseline phase in that participants were presented with a worksheet containing five problems where whole proper fractions were presented as mixed fractions less than 2.

**Interobserver agreement and procedural reliability**

To assess the percentage accuracy with which each student solved fraction problems in each phase, at least 30% of worksheets were graded by a second trained observer. The interval agreement approach was then applied to calculate the interobserver agreement (IOA). This involved dividing the number of agreements by the number of agreements plus disagreements and then multiplying the answer by 100% (Ledford et al., 2018). This yielded 100% IOA per student for all phases. The observer then gathered data on the reliability of implementing the intervention for 42% of the intervention phase for each student and procedural reliability for at least 30% of each phase. To calculate reliability (data on both procedural and implementation of the intervention), the number of steps the participant completed correctly were divided by the number of total steps and the response was multiplied by 100% (Ledford et al., 2018). This yielded 100% reliability per phase per student and 100% reliability for the intervention implementation per student.

**Social validity**

Having completed the generalisation phase, the participants were then asked a series of questions exploring their likes, dislikes, and opinions of the techniques they employed and the skills they obtained. Responses were either closed (yes/no) or open-ended. Questions assessing social validity questions were as follows: (1) Did you like the things you did in the study? What did you like/did you not like?, (2) Was it easy to learn using the materials I gave you?, (3) Would you like to continue using these strategies in the future? (4) Is there anything else you would like to tell us about your participation in the study?

**Data analyses**

To assess whether a causal relationship existed between the independent and dependent variables and the magnitude of such an effect, a visual analysis was systematically performed as this is the basis for analysing data in SCRD (Barton et al., 2018; Kratochwill et al., 2013). The procedure consisted of scrutinising data trends and stability during baseline; within-phase data trends to assess trend, level, and variability; and between-phase assessment to analyse the overlap of data, immediacy of any effect, and consistency.

**FINDINGS**

The percentage accuracy with which fraction problems were solved per student and per phase a. The visual analysis revealed the existence of a causal relationship between the independent and dependent variables. Moreover, four participants were able to generalise their ability to solve simple proper fractions to solving whole proper fractions. The mean percentage accuracy of participants' responses and standard deviation (SD) for each student per phase are presented in Table 1, along with the number of sessions each participant needed to meet the mastery criterion. Notably, all participants enjoyed making use of the intervention and felt it enhanced their learning, with the first and fourth participants in particular appreciating the intervention and expressing their willingness to engage in the sessions.

Table 1
Mean accuracy of fraction problem solving, standard deviation, and number of sessions until mastery was achieved

| Student | Baseline (SD) | Intervention (SD) | Generalisation (SD) | Number of sessions until mastery achieved |
|---|---|---|---|---|
| The first participant | 17 | 89 | 87 | 5 |
| The second participant | 42 | 100 | 0 | 2 |
| The third participant | 5 | 100 | 100 | 2 |
| The fourth participant | 20 | 85 | 86 | 5 |
| The fifth participant | 7 | 100 | 100 | 2 |

**The first participant**

At baseline, this participant exhibited 17% mean accuracy when solving fraction problems, with a low, slightly variable pattern in the data. He has fundamental misconceptions and difficulties with fractions, such as adding and subtracting fractions. While engaging in the intervention, he exhibited a mean response accuracy of 89% with a growing and variable-to-stable pattern in the data. This represents a change in mean accuracy of 72% between baseline and intervention phases with no overlapping points in the data. In the generalisation phase, he exhibited 87% mean accuracy when solving problems involving whole proper fractions. Even though this participant was more reliant on the self-monitoring checklist and fraction tiles than two of the other participants, by the final two intervention sessions, accurate problem-solving was achieved without such support. In the generalisation phase, he exhibited a stable pattern with 100% correct in the first session and 80% correct in the other two sessions see Figure 1.

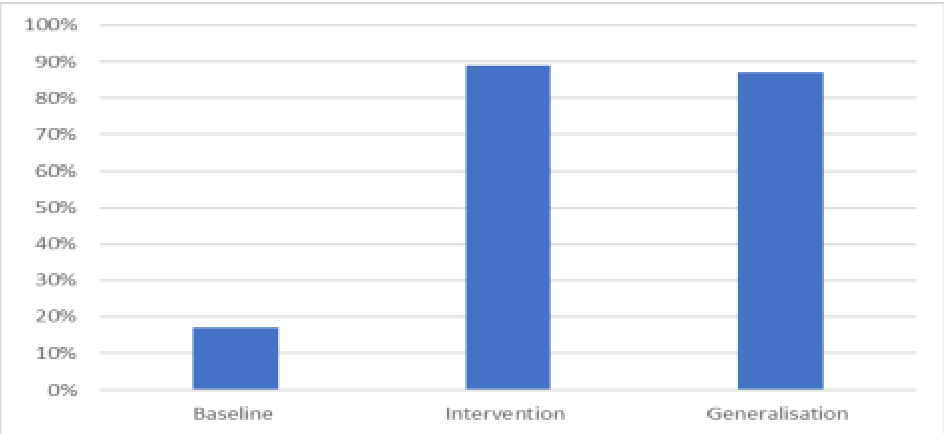


Figure1
The result of the first participant

**The second participant**

At baseline, this participant exhibited 42% mean accuracy when solving problems involving fractions, with a low, stable pattern in the data. While engaging in the intervention, his response accuracy rose to 100% straight away and did not fluctuate for five sessions in a row. This represents a change in mean accuracy of 58% between baseline and intervention phases with no areas of overlap in the data. Similar to the third and fifth participants, this participant sustained their excellent problem-solving accuracy at an abstract level without using the manipulatives and hardly ever used the fraction tiles. By contrast, he exhibited 0% accuracy when generalising his skills to solve problems involving whole proper fractions, albeit for one session only see Figure 2.

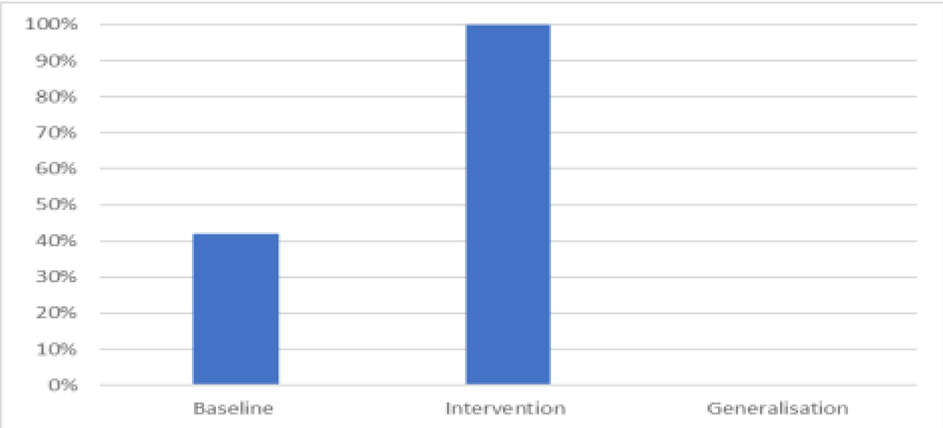

Figure 2
The result of the second participant

**The third participant**

At baseline, this participant exhibited 5% mean accuracy when solving problems involving fractions (adding and subtracting fractions) such as adding together both the numerators and then the denominators to give the incorrect answer, with a low but stable pattern in the data. When undertaking the intervention, his response accuracy rose to 100% straight away and remained at that level for five sessions in a row. This represents a change in mean accuracy of 95% from baseline to intervention phases with no points of overlap in the data. In the generalisation phase, this participant exhibited 100% accuracy in solving problems involving whole proper fractions for three sessions in a row. It was noted by the interventionist that the participant was able to sustain perfect accuracy in solving problems at an abstract level without the use of manipulatives and only used fraction tiles during the first session, abandoning them thereafter see Figure 3.

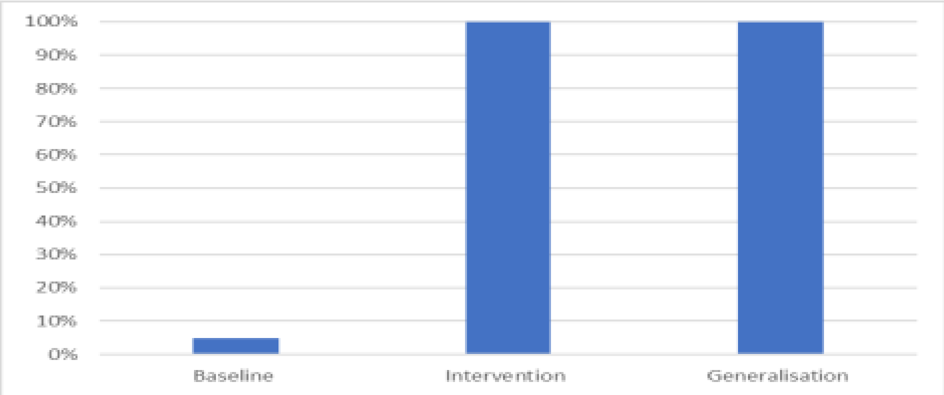


Figure3
The result of the third participant

**The fourth participant**

At baseline, this participant exhibited 20% mean accuracy when solving problems involving fractions (adding and subtracting fractions), with a low, slightly fluctuating pattern in the data. While undertaking the intervention, his response accuracy rose to 85% with a growing and variable-to-stable pattern in the data. This represents a change in mean accuracy of 65% between baseline and intervention phases with no points of overlap in the data. In the generalisation phase, this participant exhibited a mean accuracy of 86% when solving problems involving whole proper fractions. Even though this participant was more reliant on the self-monitoring checklist and fraction tiles than two of the other participants, by the final two sessions he was able to solve problems accurately without any form of support. In the generalisation phase, he exhibited a stable pattern with 100% correct accuracy during the initial session and 80% accuracy in the two sessions that followed see Figure 4.

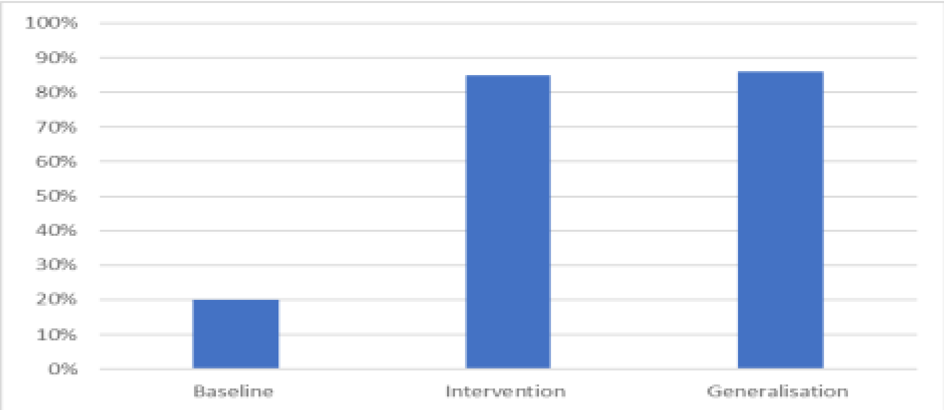


Figure4
The result of the fourth participant

**The fifth participant**

At baseline, this participant exhibited 7% mean accuracy in solving problems involving fractions, with a low but stable pattern in the data. While undertaking the intervention, his response accuracy increased to 100% straight away and remained at that level for five sessions in a row. This represents a rise of 93% in mean accuracy from baseline to intervention phases with no points of overlap in the data. In the generalisation phase, this participant exhibited 100% accuracy in solving problems involving whole proper fractions for three sessions in a row. It was noted by interventionist that this participant was able to sustain this perfect level of accuracy at an abstract level without using manipulatives and only used fraction tiles during the first session, abandoning them thereafter see Figure 5.

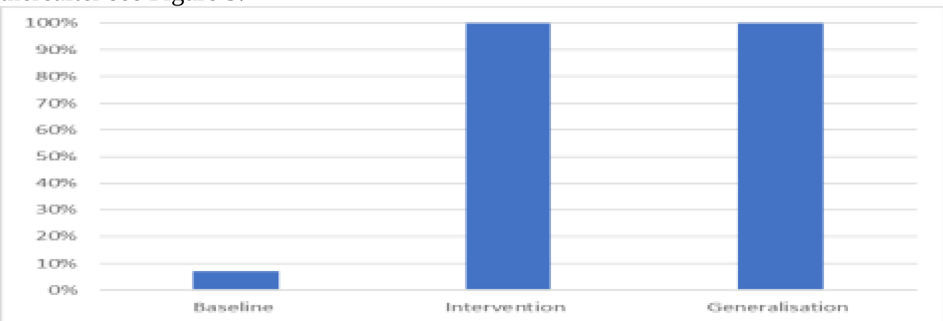


Figure 5
The result of the fifth participant

**DISCUSSIONS**

The objective of this research was to assess whether the accuracy with which five primary school students with ASD solved simple problems involving fractions would be affected by the use of VM instruction with concrete manipulatives in combination with a self-monitoring checklist and a comprehension check. In contrast to students without disabilities, such students often find it challenging to learn mathematics, particularly when it comes to learning fractions (Hecht & Vagi, 2010; Misquitta, 2011), which is an essential mathematical ability for the majority of students (Hecht & Vagi, 2010; Mazzocco et al., 2013).

The principal results indicate that all students exhibited a notable increase in accuracy from baseline to intervention sessions and four were able to generalise their skills to solve problems involving whole proper fractions. These findings align with those of Hughes (2019) who successfully taught a middle school student with ASD how to simplify fractions using VM in conjunction with a self-regulation technique. In another study, Bouck et al. (2019) reported that using virtual manipulatives in the form of a virtual-representational-abstract instructional sequence enabled a middle school student with ASD to rapidly develop the ability to identify fraction problems. In this study, the mix of techniques employed enabled students to develop the ability to solve problems involving fractions and then apply this to solve similar fraction problems, one exception being the second participant who was not able to generalise their skill in this way. The capacity of the intervention to exert such effects suggests the students in question were able to develop both a procedural and conceptual understanding of the skills required to solve fractions, which is a vital ability given the range and inherent nature of mathematics.

For instance, students need to have a solid understanding of rational numbers (e.g., fractions) if they are to succeed in most forms of higher-level mathematics. Even if they do not go on to take courses in higher education, interventions to enhance the way in which rational numbers are learnt will enhance people's quality of life as they will often come across fractions in their daily lives and need to know what they mean and how to deal with them. With respect to students diagnosed with autism, they can be assisted in living a fulfilling life post-school in domains such as employment and independent living if they are provided with appropriate pedagogical instruction (Stroizer et al., 2015; Wong et al., 2021). To cater for the disparate learning needs of students with ASD in scenarios as diverse as one-on-one intensive tuition, inclusive classrooms, and homework support, teachers will value the option of implementing an intervention comprising VM instruction and concrete manipulatives in conjunction with specific behavioural techniques.

The multi-faceted nature of this intervention means it can help students in a range of ways. First, the VM can be employed to deliver methodical and extremely clear instruction on specific topics, which has been established as an effective method for teaching mathematics. Second, to reduce the disparity in achievement between students with disparate learning needs, VM in conjunction with manipulatives and self-monitoring checklists represents a wide-ranging and detailed method for offering personalised tuition and enabling students to master specific concepts and skills before progressing to more sophisticated topics. For instance, research has revealed the positive impacts on learning mathematics offered by the use of VM, especially when applied in tandem with behavioural and academic techniques such as self-monitoring checklists, manipulatives, and the concrete-representational-abstract [CRA] framework (e.g., Hughes, 2019; Hughes & Yakubova, 2019). This enables teachers to focus on enhancing specific skills and reduce gaps in the mathematical knowledge possessed by students. In the current study, all students developed the capacity to solve problems involving simple proper fractions and then progress from a concrete problem-solving phase to one that is more abstract in nature. This suggests that concrete manipulatives may assist students in developing an enhanced understanding of the use of abstract mathematical concepts (Maccini & Gagnon, 2000; Root et al., 2017). Moreover, the fact that VM delivers pedagogical content in a methodical and visual way using explicit and invariable language while ensuring students concentrate on the topic in hand means it is especially efficacious in teaching a broad suite of skills to students diagnosed with ASD (Hughes et al., 2016; Hughes and Yakubova, 2019; Yakubova et al., 2015). The intervention is structured to permit variations according to need and quantity; hence additional sessions and opportunities would have enabled the second participant to practice and master the skills they did not acquire in this study.

In short, the use of explicit instruction enshrined in the CRA framework entails the educator modeling and thinking aloud to clarify the mathematics involved, directing the student as they strive to solve a particular problem, and then permitting the student to solve the mathematical problem without any support. Its pedagogical basis means that the CRA framework is an approach supported by decades of research (e.g., Agrawal & Morin, 2016; Bouck et al., 2017; Bouck & Park, in press; Butler et al., 2003; Flores, 2010; Jordan et al., 1998; Miller & Mercer, 1993; Underhill, 1977) that is recommended for students diagnosed with high-incidence and low-incidence disabilities (Browder et al., 2012; Doabler & Fien, 2013; Gersten et al., 2009; Root et al., 2017).

## LIMITATIONS

Like all experimental studies conducted in the setting of a school, this research has a number of limitations that need to be addressed. First, it was conducted only in government primary schools in the eastern region of Saudi Arabia. The results may therefore not be generalisable to the country as a whole, Nevertheless, Maxwell (2005) contends that an inability to generalise constitutes a strength of qualitative research as it focuses on particularised yet important findings. Regardless, the city was considered an ideal location for this study by the researcher as it has a large population comprising residents from all areas of the country. This was evident in the sample of participants, who came from regions as diverse as the western, north-western part, south-western, and central areas of the KSA. A second limitation was the sample of the study was purely comprised of five students due to the challenges faced with obtaining permission to expand the study to a wider students. However, the researcher needed this rich information to get an in-dept informations of these five students, otherwise it would have been difficult to reach the research objectives which led the researcher to answer his research questions. The third limitation the difficulty of adhering to the school schedule, as sometimes students were absent due to illness or visits to the doctor, and there were days when the scheduled support period for mathematics did not take place as students where following an alternative schedule. However, the latter only occurred on three occasions and did not appear to affect the results or impede students' ability to acquire the skills needed to solve simple problems involving proper fractions with 100% accuracy. The fourth limitation is that it was not possible for the male researcher to access a female cohort as students in these grades are taught in single sex classes by a teacher of the same sex.

With regard to avenues for future research, it was clear that one of the five students was not able to generalise the skill they had acquired and solve problems involving whole proper fraction problems. This was because it was the end of the school year and they only attended one session. This aligns with research that suggest students with ASD require more systematic and intensive instruction in order to generalise (Hume et al., 2009). Future research should focus on determining whether more complex fraction problems can be taught to students with ASD using VM and concrete manipulatives in conjunction with self-monitoring techniques.

## CONCLUSIONS

The results of this research add to the body of knowledge and practice on teaching students with ASD to comprehend fractions and solve simple problems involving proper fractions using VM in tandem with a comprehension check, concrete manipulatives, and a self-monitoring checklist. The results revealed that from baseline to intervention, all five students solved simple proper fraction problems with greater accuracy and four were able to apply this ability to solve problems involving whole proper fractions. This mean that using VM in conjunction with concrete manipulatives and self-monitoring checklists can be an efficacious way to deliver clear methodical teaching using invariable pedagogical language to students with ASD, thereby enabling them to develop the mastery and conceptual comprehension required to solve problems involving fractions. Moreover, the ease with which VM can be created and applied renders it a versatile way of offering comprehensive, personalised, and bespoke support that addresses the learning needs of individual students in disparate classroom scenarios.

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