International Journal of Instruction *October 2023 ● Vol.16, No.4*
e-ISSN: 1308-1470 ● www.e-iji.net *p-ISSN: 1694-609X*
pp. 17-36

ZArticle submission code: 20220913155105
Received: 13/09/2022
Revision: 04/03/2023
Accepted: 27/03/2023
OnlineFirst: 01/07/2023

Mathematical Connections Associated with the Exponential and Logarithmic Functions Promoted in the Mathematics Curriculum

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This article aims to identify the general mathematical connections and those related to exponential and logarithmic functions, promoted by the Colombian intended mathematics curriculum. In this research, Content Analysis was used, and the official and institutional Colombian intended mathematics curriculum (of 5 schools) at the middle and high school level were used as a source of information. In Colombian, the official curriculum is made up of three official documents and to select the institutional curriculum, convenience sampling was used. Among the results, we find that, although the Colombian intended curriculum assumes the idea that making mathematical connections contributes to the development of understanding and promotes some of these (feature, meaning, different representations, and modeling), the reversibility connection is not identified — which we consider central — between exponential and logarithmic functions, which shows that the teaching of these is encouraged separately. This can influence what occurs in the classroom and, therefore, is one of the possible causes of the difficulties presented in students when solving tasks related to both functions. In this sense, it is necessary that institutions consider that making mathematical connections contributes to the understanding of students and therefore must be considered when designing the curriculum. It is necessary to promote the mathematical connection of reversibility between the functions that are addressed in this study and between the objects with which it is possible to make inverse relationships.

Keywords: mathematical connections, mathematics curriculum, exponential function, logarithmic function, mathematics education

Citation: Campo-Meneses, K. G., García-García, J., & Font Moll, V. (2023). Mathematical connections associated with the exponential and logarithmic functions promoted in the mathematics curriculum. *International Journal of Instruction*, *16*(4), 17-36. https://doi.org/10.29333/iji.2023.1642a

INTRODUCTION

The exponential and logarithmic functions, according to the Ministry of National Education (MEN, 2006 [By its initials in Spanish MEN]) are concepts considered in the Colombian mathematics curriculum in middle school (specifically in grade 9) and in high school (in grade 11). In addition, according to the specialized literature, they are important mathematical concepts for learning of several physical and social processes; they are used for modeling several phenomena from real life and other sciences and play a central role in the university's Mathematics courses, such as Calculus, Differential Equations, and Complex Analysis, however, they represent a difficult topic for students (Weber, 2002).

This latter has led several researchers (Campo-Meneses & García-García, 2020; Castro et al., 2017; Ferrari-Escolá et al., 2016; Kuper & Carlson, 2020) to focus their interest on analyzing or promoting student learning (from secondary to a higher level) when they work with situations associated with the exponential and logarithmic functions that are proposed to them. In general, research on exponential and logarithmic functions has reported that students continue to show different difficulties in understanding them. For example, the following are reported: university students do not relate exponential laws to logarithmic laws (Weber, 2002); they do not develop a high level of covariational reasoning (Ferrari-Escolá et al., 2016); there is a preference when working with the natural logarithm function as an inverse of the natural exponential, since its manipulation is easier, greatly limiting the general use of this function in algebraic contexts (Escobar, 2014) and university students have difficulty making mathematical connections between these functions (Campo-Meneses & García-García, 2020).

These difficulties have been reported in different countries, one of them in Colombia (Campo-Meneses & García-García, 2020; Campo-Meneses & Cruz, 2020; Escobar, 2014). The difficulties described are evidence, in some way, of the lack of understanding of the students regarding the exponential and logarithmic functions, which may be a consequence of the curriculum taught by the teacher, which is guided by the intended curriculum (official and institutional), since teachers use this to plan their classes (see Figure 2). Therefore, the importance of analyzing the intended curriculum and identifying the teaching that is being promoted about these functions.

This analysis is possible to make using the mathematical connections, since we consider that identifying those that are promoted in the teaching of these functions could give evidence of what students are intended to understand. This is because the literature in Mathematics Education affirms that making mathematical connections contributes to the development of understanding and analyzing the connections that a subject makes allows inferring their level of understanding (Bingölbali & Coskun, 2016).

For example, if the reversibility connection (which we consider as a central connection) between exponential and logarithmic functions is promoted, it could contribute to decreasing the difficulties about the relationship between logarithmic and exponential laws. In addition, it would require the prior make of other types of mathematical connections that allow the student to better conceptualize these functions and make different relationships between their elements. All this would contribute in some way to reduce some of the difficulties reported in the literature.

In that sense, according to García-García (2019) and Campo-Meneses and García-García (2020), it is necessary to carry out research that focuses on studying the mathematical connections associated with the exponential and logarithmic functions promoted by the teacher or the official curriculum. Thus, this research aims to answer the following question: what mathematical connections, both general and related to exponential and logarithmic functions, does the Colombian intended mathematics curriculum?

Studies have been carried out on the curriculum from different approaches (Dolores, et al., 2020; Dolores & Mosquera, 2022; Francis, et al., 2020; Gök, et al., 2019), however studies of mathematical connections that are promoted in this (García-García, et al., 2022) are scarce and even more about those connections that are promoted in the Colombian mathematics curriculum. This is one of the reasons why this research is pertinent. In addition, according to Hume and Coll (2010), the intended curriculum influences what happens in the classroom. We believe that the mathematical connections that are identified in the intended curriculum reveal an overview of those connections that are possibly promoted in the classroom. Likewise, this allows us to recognize how the teaching of exponential and logarithmic functions is conceived in the light of mathematical connections and thus in future works make proposals that affect this field and contribute to decreasing the difficulties reported in the literature.

Thus, in this research, three official documents were taken as a source of information, which make up the official mathematics curriculum, and five institutional curriculums were selected by convenience sampling.

Conceptual Framework

Mathematical Connections (MC)

Mathematical connections are assumed as true relationships between mathematical ideas, theorems, procedures, concepts, etc., with real life or with other disciplines (García-García & Dolores-Flores, 2018). So, if in the text we directly find the word connection or phrases that refer to true relationships, we will say that some mathematical connection is being promoted. The typologies of mathematical connections, according to the focus of this research, are described below according to García-García and Dolores-Flores (2018) and Businskas (2008):

•Feature (F): is the relationship between a mathematical concept and its invariant attributes that distinguish it from others.

•Different representations (DR): it is the relationship between two different semiotic representation referring to the same mathematical concept, but also the relationship between two different ways of representing this concept in the same register.

•Part-whole (PW): it is the logical relationships established between mathematical concepts performed by students. These can be generalization or inclusion.

•Procedural (P): is of the form A is a procedure used when working with object B. In this case, the mathematical connection includes all the rules, algorithms, or formulas established within a semiotic register and that are used to find a result.

•Meaning (M): this can be seen in two ways: as the relationship between the concept and the definition or as the relationship between the concept and its contexts of use.

•Reversibility (R): it is the bidirectional relationships between two mathematical operators, that is, someone can start from a concept A to reach B and in turn reverse the process starting from B to return to A.

•Implication (I): this is of the type A implicates B, an object logically leads to another, and they tend to be written in the form if . . . then . . .

•Instruction-oriented (IO): it is the relationship between the knowledge that the student is going to learn or is learning and the knowledge that he has (previous), or the relationship between the set of concepts and procedures (which are prerequisites for learning another topic) and the new topic.

•Modeling (Mo): it has to do with the relationship between the world of mathematics and the real one (or the daily life of students) and between mathematics and other sciences.

Mathematical connections to promote understanding of exponential and logarithmic functions

As mentioned, research in Mathematics Education reports that mathematical connections play an important role in understanding, as making them contributes to the understanding of a concept by a person and allowing him or her to develop other mathematical skills. So, to study the mathematical connections that a person makes allows to infer his or her level of understanding (Bingölbali & Coskun, 2016; Mhlolo, 2012). In this sense, making mathematical connections is considered a condition (and in turn evidence) for understanding (Campo-Meneses & García-García, 2021; Campo-Meneses et al., 2021).

According with this, it is important to introduce the term central mathematical connection proposed by García-García and Dolores-Flores (2018), who characterized it as one that is directly linked to the object of study and that its make is evidence, of the understanding of the concept by the person and requires in turn, making the other mathematical connections. This mathematical connection is characterized because it allows deriving other mathematical connections once it has been made.

In this sense, for the case of exponential and logarithmic functions, the reversibility connection is considered as the central one since they are inverse functions. Making of this mathematical connection requires having made the mathematical connections of different representations, meaning, part-whole, feature, and procedural with each function. Likewise, making the reversibility connection produces the implication connection as a consequence (Campo-Meneses et al., 2021) and influences the making of the modeling connection, as shown in Figure 1.

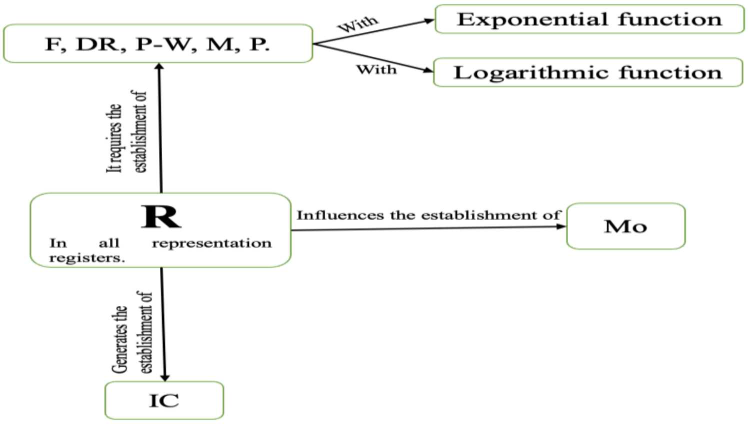

Figure 1
Central connection with respect to exponential and logarithmic functions

Description of the Colombian curriculum

The curriculum is understood as any activity that allows organizing and describing a training plan and that aims to answer questions regarding the nature of the knowledge to be taught, learning, teaching, and the usefulness of such knowledge (Gómez, 2002).

According to Valverde et al. (2002), there are four kinds of curriculums: the intended, which establish sets parameters for what students should learn and how to organize that learning; potentially implemented, which is composed of the textbooks and other organized recourse materials; implemented, which refers to what is actually taught in the classroom, who teaches it and how it is taught (strategies and practical activities) and the attained, referring to what the students have learned. For this reason, according to the way in which education is regulated in Colombia, we assume that the intended curriculum includes both the official national documents (hereinafter we will refer to as the official curriculum) and the particular documents of each institution (institutional curriculum).

In the same way, the statements on national curricular policies are designed to be interpreted by teachers in order to assist them in the planning of learning experiences (Gómez, 2002; Hume & Coll, 2010). The official curriculum influences the implemented curriculum, although they do not align perfectly with what teachers interpret (Hume & Coll, 2010), and this can be reflected in the process that is followed to move from the official curriculum to the implemented one (see Figure 2).

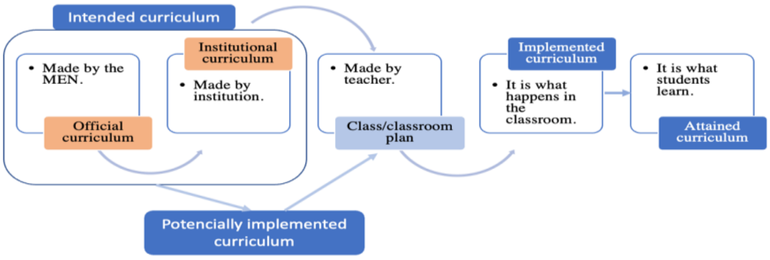

Figure 2
Process on the curriculum in Colombia
Note. The official and institutional curriculums make up the intended curriculum.

In this sense, in Colombia, the official mathematics curriculum is made up of three documents (see Table 1), which are the basis for the elaboration of the institutional curriculum (also called study plan). The latter is prepared by each educational institution since according to the General Education Law 115 (MEN, 1994) the institutions enjoy curricular autonomy according to their needs or guidelines, but they must be based on official documents. Thus, the official documents are: Curricular Guidelines (CG), Basic Standards of Competence in Mathematics (BSCM) and Basic Learning Rights in Mathematics (BLRM), which are prepared by the MEN.

Table 1
Official documents that make up the official Colombian mathematics curriculum

Document	¿What is it?
Curricular Guidelines (MEN, 1998)	Document prepared by the MEN that describes in a general way the objectives of Colombian education.
Basic Standards of Proficiency in Mathematics (MEN, 2006)	Document that is based on the Curricular Guidelines and describes the competencies that students must develop.
Basic Learning Rights in mathematics (MEN, 2016)	Document based on the Curricular Guidelines and the Basic Competency Standards which raises the elements to build teaching routes that promote the achievement of learning year by year so that, as a result of a process, students reach the standards proposed by each group of grades.

As mentioned previously, each school or institution is free to design its curriculum. So, Table 2 describes the institutional curriculum of five Colombian schools that are the object of analysis in this research. It should be noted that since it is not possible to reveal the information of the schools, the selected institutional curriculum is not referenced.

Table 2
Description of the institutional mathematics curriculum of five Colombian schools

School	Description of the institutional curriculum
School 1	It is structured by grade level, detailing the standards, Basic Learning Rights, components, themes, competencies, learning and evidence.
School 2	It is structured by grade level, detailing the standards, Basic Learning Rights, components, competencies, performance indicators, performance levels, teaching units and evaluation criteria.
School 3	It is structured by grade level, detailing the standard, Basic Learning Rights, learning sequences, evidence of learning, situations that promote learning and strategies for evaluation.
School 4	It is structured by school grade, detailing the standards, the performances that are related to the Basic Learning Rights and thematic axes to work on.
School 5	It is structured by grade level, detailing the standard along with Basic Learning Rights, performances, subtopics, period, and weeks.

METHOD

This is qualitative research. The Content Analysis proposed by Bardin (2002) was used as a method that is structured in three phases.

Phase 1: Pre-analysis

In this phase it was reviewed how the Colombian intended mathematics curriculum was formed and it was identified that, currently, there are three official documents (they make up the official curriculum) on which the institutions are based to design their study plans (institutional curriculum).

Due to the curricular freedom that institutions have, in addition to official documents, it was necessary to select institutional curriculums, to complement the information about the mathematical connections that are promoted in the teaching of exponential and logarithmic functions. This, because the teachers to design their classes are usually based on both the official and the institutional curriculum and, these two make up the intended curriculum, which is the object of analysis of this research.

The sample of the five institutional curriculums was selected through convenience sampling (Creswell, 2013), where the information was provided by teachers since it is not possible to easily access the school curriculums. For this, a questionnaire was applied where the following was mainly asked: In which institution do you work? Does the institution have a mathematics curriculum or study plan? Does the institution allow access to this curriculum or study plan? Thus, the documents that make up the official curriculum and the five institutional curriculums were the source of information for the analysis.

Phase 2: Material exploration

For the purposes of this research, sentences that referred to the making mathematical connections according to the definition (described in the conceptual framework) or that the word connection directly appeared were identified in the documents under analysis.

The first case corresponds to implicit connections— the mathematical connection is inferred from the extracts as it is not explicit— and the second to explicit connections. Afterward, it was identified which type of mathematical connection was involved. It is important to remark that more than one type of mathematical connection can be identified in the same extract.

For example, in the Curricular Guidelines (MEN, 2006) for the area of mathematics we find the following paragraph:

> Communication plays a fundamental role, helping children to build the links between their informal and intuitive notions and the abstract and symbolic language of mathematics; It also plays a key role in helping students make important connections between physical, pictorial, graphic, symbolic, verbal, and mental representations of mathematical ideas (p. 74).

The phrase *connections between physical, pictorial, graphic, symbolic, verbal, and mental representations of mathematical ideas*, refer to the mathematical connection of different representation types.

Phase 3: Treatment and interpretation of the results obtained

To carry out this phase, the data was triangulated between researchers working on the line of mathematical connections and the analysis was drawn up. This was done as follows: each researcher carried out the analysis separately and then met to discuss what was found. When there were differences in the categories established by the researchers, then we discussed based on the data and reached agreements based on the arguments that supported them. The analysis ended when a consensus was reached on the results found from the analysis and discussion made.

FINDINGS

This section is presented in three groups, it begins by talking about the mathematical connections identified in the official curriculum, then those identified in five institutional curriculums are described and, finally, the results of the connections promoted in the official curriculum coincide with those promoted in the institutional curriculum.

Mathematical connections promoted in the official Colombian curriculum

General mathematical connections identified

The content analysis of the official curriculum shows that six types of mathematical connections are generally promoted (see Table 3). However, of the three documents that comprise it, only two (the Curricular Guidelines and the Standards) promote general mathematical connections.

Table 3
Example of general mathematical connections promoted in the official curriculum

MC	Extract example	Description	Document	Implicit/explicit
				Explicit
R	Communication plays a fundamental role […] it also plays a key role in helping students to draw important connections between physical, pictorial, graphic, symbolic, verbal and mental representations of mathematical ideas.	Connections between different representation.	CG	Explicit (E)
I O	Students in interaction with the teacher and in cooperative dialogues among themselves, make connections between what they previously know and what is new	Connection between what students previously know and what is new.	CG	Explicit (E)
P	Understanding of the relationships between the context of the problem and the necessary calculation.	Connection between the problem and the calculation necessary to solve it.	CG	Implicit (I)

As can be seen in Table 3, different representations connection was identified in the Curricular Guidelines with the highest frequency, which appears explicitly, however, other mathematical connections expected appear both explicitly and implicitly. Regarding this, the use of five representations of mathematical concepts is promoted: physical, pictorial, graphic, verbal, and symbolic, in which relationships between them are made. Students are expected to make these connections as they communicate mathematical ideas. However, tabular representation is not included, which is important when working with functions or other mathematical concepts.

The instruction-oriented mathematical connection is a connection that is considered important in the Curricular Guidelines (see Table 3), since the premise is that, if students manage to connect the new knowledge with what they already know, it is because learning is being generated. This connection appears explicitly, is quite general, and is expected to be made by students by working with any mathematical object at all academic levels.

The reversibility connection is promoted between operations, even in the Curricular Guidelines it is stated that "the inverse relationship between operations is another valuable connection that provides the student with another way of thinking about the problem" (MEN, 1998, p. 34). This connection appears explicitly in the Curricular Guidelines (see Table 3) and they focus it mainly when working on basic operations, there is no evidence that it is promoted in different aspects than problems that include inverse operations of addition and multiplication. However, this can be interpreted with other operations such as radicals, potentiation, and logarithm, but not with the work of inverse functions.

On the other hand, the procedural mathematical connection is promoted in two ways: between operations and between the problem and the calculations necessary to solve it. The first (explicitly) is expected to occur when a student solves a problem and manages to realize that he has more than one operation that he can use to solve it and that these produce the same result and, the second (of implicitly) can be made when the student employs a procedure that uses properties, rules, and operations that relate to the problem at hand.

Regarding the mathematical modeling connection, it is explicitly promoted between mathematics and real life and between mathematics and other disciplines. This can be seen in the following excerpt:

> The context of learning mathematics is the place […] from where connections are made with the daily lives of students and their families […] and, in particular, with other sciences and with other areas of mathematics itself (MEN, 2006, p. 70).

Both the Curricular Guidelines and the Standards emphasize that mathematics should be useful for daily life, which is why they consider it important to make the connection between the contents of mathematics and what the student experiences in their daily lives. Finally, the connection of meaning is promoted implicitly between the mathematical content and the meaning attributed to it, in the same way between the content and its meaning, but it is only promoted by the Standards.

Mathematical connections related to exponential and logarithmic functions

Before focus on the mathematical connections identified in the official curriculum, it is important to point out what standards and learnings are intended to be developed in the teaching and learning of these functions in each document. In the Basic Standards of Competence in Mathematics, it is made that between the eighth and ninth grades, students must be able to develop the following standard: “I analyze in Cartesian graphical representations the behavior of change of specific functions belonging to families of polynomial, rational, exponential functions, and logarithmic” (MEN, 2006). While in the Basic Learning Rights (MEN, 2016), they place the teaching of these functions in the tenth and eleventh grades, where the following two Basic Learning Rights are identified respectively (one for each grade, and each one with its respective learning evidence):

> A student solves problems using properties of functions and uses tabular, graphical, and algebraic representations to study variation, numerical trends, and rates of change between magnitudes (p. 77).
>
> A student uses properties and functional models to analyze situations and to make functional relationships between variables that allow the study of variation in intra-school and extra-curricular situations (p. 85).

Although the exponential and logarithmic functions are not specifically identified in these rights, they are assumed to include all functions by the provisions of the Standards.

According to the above (and the evidence of learning of each Basic Right and of the learning), it was identified that the official curriculum promotes the making of five typologies of mathematical connections related to the exponential and logarithmic functions: feature, meaning, different representations, procedural and modeling, where the last four are consistent with the previously identified general mathematical connections (although not necessarily in the same way). Table 4 shows the typologies of mathematical connections identified, some examples of extracts where the typologies were identified, the code that corresponds to how the connection is made, and the document where they are promoted.

Table 4
Mathematical connections associated with the exponential and logarithmic functions promoted in the official curriculum

MC	Code	Document	I/E
M	(1) Relationship between the function and the contexts in which it appears. (2) Relationship between the function and its meaning.	CG	I
F	(1) Relationship between the sets in which the function is defined.	CG	E
	(2) Relationship between the graphical characteristics of the function.	All	I
	(3) Relationship between the symbolic and graphical characteristics of the function.	BLRM	E
	(4) Relationship between the properties of each function.	BLRM	I
DR	(1) Between a symbolic representation and a graphic representation.	All	E
	(2) Between two graphical representations.	BSCM	E
P	(1) Between the problem and the procedure to solve it.	BLRM	I
Mo	(1) Between the problem and the functional model.	BLRM	E

The feature mathematical connection is promoted in the three official documents in different ways (see Table 4). It is particularly promoted in the algebraic and graphic registers; it is also identified when it is stated that the student must relate the sets in which each function is defined and also the properties of each one. An example of this connection is found in the only standard proposed in the Standards, which evidence, for these functions that the student between the eighth and ninth grade of secondary school must be able to analyze the behavior of change in the graphical register, which implies that it characterizes the behavior of each function and relates it to the particular function.

Regarding the connection of meaning, this is promoted only in the Curricular Guidelines (see Table 4), and in the two senses proposed by the conceptual framework. For its part different representations connection is promoted in the three documents and most explicitly, but only between a symbolic representation and a graphic representation, however in the Standards this connection between two representations in the same register is also promoted (graphic). For instance, it states that students must be able to express and translate between verbal, graphic and symbolic languages, this implies relating a verbal representation with a graph, a verbal with a symbolic, and a graph with a symbolic (previously mentioned).

The Basic Learning Rights also promote the procedural mathematical connection about the functions under study, similarly, as is evidenced by the Curricular Guidelines in the general connections, although only between the problem and the procedure to solve it (see Table 4). Likewise, these two documents also promote the mathematical modeling connection, only that the Basic Learning Rights do it between the problem and the functional model.

Mathematical connections promoted in the institutional curriculum

After analyzing the five institutional curriculums, we identified that two of the five schools explicitly evidence the standard or Basic Learning Rights related to the teaching of these functions as presented in the official curriculum. Furthermore, only two of them (school 1 and school 2) promote the making of general mathematical connections. The two curriculums promote the modeling connection between the mathematical object and real life (school 1 implicitly). School 2 also promotes this connection between mathematics and other sciences.

The procedural mathematical connection is implicitly promoted by school 1 between the problem and the calculation necessary to solve it. For its part, school 2 promotes making the part-whole mathematical connection in an implicit way between a particular case and the general mathematical content, and the connection of different representations between different representations of an object.

Regarding the mathematical connections associated with the exponential and logarithmic functions, school 1 places the teaching of these functions in the tenth grade, the Standards and Basic Learning Rights that they adopt are not related to the functions under study. However, they propose two learning evidence related to such functions where the making of the connection of meaning between the function (exponential or logarithmic) and the context of use is promoted, or between the function and its definition, as shown in Evidence One: "I understand the definition of the logarithmic and exponential function". It should be noted that this (implicit) connection is identified in this evidence because, for the student to understand the definition, he must make a connection between the function and the definition.

In school 2, the teaching of these functions is located in the ninth grade, and although the standard related to these functions appears explicitly, at the time of development it is only reported up to the quadratic function.

In school 3, the teaching of these functions is located in ninth and in the eleventh grade. For the teaching in ninth grade, they adopt a standard and two Basic Learning Rights that are not directly related to these functions, however, these are addressed as part of the sequences proposed for this grade of study and in the learning evidence that they propose are identified the following mathematical connections:

Meaning connection (implicitly), between the function and the contexts of use and, between the function and its meaning since students are expected to recognize an exponential or logarithmic function and identify and use them in different contexts. The feature connection between the function and the characteristics that describe it in the different representation registers, and between the function and its properties implicitly. An example of this connection is found in the extract: “Analyzing characteristics of exponential and logarithmic functions”, where analyzing involves relating each function to its characteristics.

For the part-whole mathematical connection (explicitly), between the family of functions and the particular cases, inclusion is privileged, since it is expected that students will be able to give examples of functions that meet the characteristics of being exponential or logarithmic. The connection of different representations (explicitly) is promoted between symbolic representation and a graph. In addition, the maki of this connection between different symbolic representations is promoted, but only for the logarithmic function. It should be noted that the making of other representations necessary to address the functions under study is not encouraged and the transition from symbolic to graphic expression is privileged.

The procedural mathematical connection is promoted between the operations or algorithms referred to the logarithmic function. This mathematical connection is promoted implicitly and is only promoted in the symbolic representation, privileging the relationship between the problem and the calculation necessary to solve it.

For the eleventh grade, the making of the same mathematical connections promoted in the ninth grade is promoted, but they delve into the mathematical connections: feature when working characteristics of the base and its relationship with the graph, and, part-whole when working in detail exponential functions and natural logarithm. In addition, two more connections are added: modeling between each function and real-life situations, and reversibility between the two functions specifically in the symbolic and graphic representations. An example of this last connection is identified in the evidence “Recognize the passage of expressions from exponential to logarithmic form and vice versa in solving equations that include this type of expressions”, in which it shows the reversibility connection in the symbolic representation to the go from an exponential expression to a logarithmic one and vice versa.

In school 4, the teaching of these functions is identified in the ninth grade and the standard related to them is assumed, however, only one thematic axis is identified for the study of the classification of functions where the exponential and logarithmic functions appear, but it is not detailed how much of these functions will work. For this reason, it is said that this curriculum does not show the connections that they intend to promote with the teaching of these functions.

Finally, in school 5, the teaching of these functions is identified in ninth grade, but it does not adopt the Standards and Basic Learning Rights in a textual way, but they make an adaptation. In line with this, it is observed that they promote three types of mathematical connections: of meaning (implicitly), between the function and its definition, since it is expected that the students will be able to associate and understand the definition of each function; feature (explicitly), between the function and its main characteristics, for which it is expected that students make a relationship between the properties of each function in different representations, and the mathematical connection of different representations (implicitly), between different representations such as is shown in performance: “student explores the different ways of representing a function (tables, graphs, etc.) since exploring the different ways of representing an exponential or logarithmic function implies making connections between these representations, which allows identifying characteristics and performing the step from one representation to another”.

Comparison between the connections promoted in the official curriculum and those promoted in the institutional curriculum

Considering the analysis of the general connections and those associated with the functions under study that are promoted in both the official and institutional curriculums, a comparison was made between what these curriculums show. Regarding the general connections (evidenced only in schools 1 and 2), the institutional curriculum is consistent with the official one in promoting the modeling connections between mathematics and real life and other sciences, procedural between the problem and the calculation necessary to solve it and the different representations connection. Unlike the official curriculum, school 2 also promotes the part-whole connection between the particular case and the generality. Regarding the mathematical connections related to the functions under study, Table 5 shows a summary of the connections promoted by the official curriculum (described in Table 4) and which of these were identified in the institutional curriculum of the 5 schools.

Table 5
Comparison between the mathematical connections associated with the exponential and logarithmic functions promoted in the official curriculum and in the institutional curriculum

MC-Official curriculum (Table 4)	MC promoted by institutional curriculums				
	School 1	School 2	School 3	School 4	School 5
M_1	●		●		
M_2	●		●		●
F_1, F_2, F_3, F_4, DR_1			●		●
DR_2					●
P_1, Mo_1			●		

According to Table 5 and the previous analysis, schools 2 and 4 did not show promoting mathematical connections for exponential and logarithmic functions. The connections that are promoted in schools 1, 3 and 5 are contemplated in the official curriculum, while school 3, although it promotes some of the connections of the official curriculum, promotes three other important connections that were not identified in the official curriculum: procedural connection between the algorithms used to solve problems, the reversibility connection between these two functions, and the part-whole mathematical connection between a particular case and the generality.

DISCUSSION AND CONCLUSION

The aim of this research was to identify the mathematical connections, both general and related to the exponential and logarithmic functions, promoted by the Colombian intended mathematics curriculum. To achieve this, the content analysis was used and three official documents that make up the official mathematics curriculum and five institutional curriculums selected through convenience sampling were used as a source of information (Creswell, 2013). This was done because, to analyze the mathematical connections promoted by the Colombian intended mathematics curriculum, it was not enough to analyze the official curriculum since in Colombia each school has the freedom to design its curriculum, taking the official curriculum as a reference. In this sense, to know which connections are being promoted, it was necessary to analyze the documents that make up the intended curriculum, as indicated in the reference framework.

Thus, it was identified that the general mathematical connections promoted in the official curriculum are not the same as those promoted in the institutional curriculum of the selected schools (see Table 5), with those identified in the institutional curriculum being minimal, and even only in two of the five schools, the making of general connections is promoted. In this sense, the official curriculum promotes making six typologies of general connections (see Table 3): mathematical connections of meaning, instruction-oriented, reversibility, procedural, modeling, and different representations, of which only the last three are promoted in the curriculum of the selected schools, adding the part-whole mathematical connection.

This result is similar to that reported in García-García et al. (2022) who identified the procedural type connections, different representations, modeling, meaning, part-whole and characteristic in the Mexican mathematics curriculum, however the two findings differ in the feature and instruction-oriented connections. It should be noted that the types of connections that are promoted may vary depending on the focus of the curriculum and even the institution, as reported in this research.

It should be noted that the reversibility connection (the central connection between these functions) is only promoted between operations in the official curriculum, mainly between elementary ones, which would not cover the entire treatment of functions. Furthermore, feature and implication connections are not generally promoted in the intended curriculum.

Regarding the exponential and logarithmic functions, their teaching is not located at the same academic level according to the official curriculum, since in the Standards ranks in grade eight-ninth and in Basic Learning Rights in grade tenth and eleven. Similarly, in the institutional curriculum of the five selected schools, in schools 2, 4, and 5 it is ranked ninth, in school 1, in the tenth, and school 3, in ninth and tenth grade. This may be due to the lack of correspondence between the official documents regarding the grade to which these functions should be taught and even regarding what is covered of these and the mathematical connections that should be promoted.

The official curriculum promotes making six typologies of general connections, of which only four coincide with those related to the exponential and logarithmic functions. The general connections promoted in the institutional curriculum of the two schools coincide with the particular ones for the functions under study and there are even more connections related to these functions.

According to Table 5, only the curriculum of three of the five schools' evidence that it promotes the making of mathematical connections related to the exponential and logarithmic functions, and in general, these connections coincide with those promoted by the official curriculum, although not to the same extent. In addition, one of the schools in its curriculum promotes making other connections.

This shows that some of the selected schools, in their institutional curriculum, are not considering what is stated in the official curriculum about the exponential and logarithmic functions, for the development of the units. Some schools consider some elements and others cover more than what the official curriculum itself proposes. This is evidenced in Gómez et al. (2016) since they affirm that the relationship between official documents and the study plan or institutional curriculum does not necessarily occur in Colombia, which is also reported in Dolores and Mosquera (2022).

Among the mathematical connections related to the functions under study, which are not evident in the official curriculum and the 5 institutional curriculums, is that of reversibility. This is only promoted by the school's curriculum 3, which is evidence that in general, the Colombian intended mathematics curriculum promotes the teaching of these functions separately and this in practice has been a problem for students to understand these functions since they are inverse functions, the reversibility connection is the most important when working with these functions (Campo-Meneses & García-García, 2020).

In this sense, it is considered necessary to review a bigger sample of institutional curriculums, to analyze if something similar to that found in this research occurs and thus be able to thoroughly investigate the relationship between the official and institutional Colombian mathematics curriculum and identify how it is can contribute, as this is outside the scope of this paper.

According to the results of this research, it is necessary both in the official curriculum and in the selected institutional curriculums to promote more mathematical connections (mainly that of reversibility), in such a way that they contribute to the development of student's understanding, since the official curriculum assumes the idea that connections are important for students to understand, which is reported by various investigations in the line of mathematical connections (Bingölbali & Coskun, 2016; Campo-Meneses & García-García, 2020; Mhlolo, 2012). All this because, according to Hume and Coll (2010), what is addressed in the intended curriculum influences the implemented curriculum, and therefore in this way, it could be possible to minimize some difficulties related to the student's understanding of these functions that the literature reports (Campo-Meneses & García-García, 2020; Ellis et al., 2016; Ferrari-Escolá et al., 2016; Gruver, 2018; Weber, 2002).

This research carried out is inserted in one of the scenarios (curriculum review) to investigate in the line of mathematical connections, by the approach of García-García (2019) and Campo-Meneses & García-García (2020) and shows an overview of the mathematical connections promoted in the Colombian curriculum, which had not been done until now and therefore is a contribution both to the line of mathematical connections and to the panorama related to the teaching-learning of the functions studied. However, one of the limitations of this research is that the results related to the curriculum of the five schools cannot be generalized because it is a case study where the sample is selected for convenience. It is known that in order to broaden this panorama, a larger sample would have to be selected using mixed methods that allow having a representative sample of all Colombian schools.

Finally, a future investigation could use the same sample of selected schools, analyze the class plan of some mathematics teachers and the classes that they allocate for teaching the exponential and logarithmic functions and compare them with the results obtained in this investigation. Likewise, these results can be used so that in subsequent research more institutional curriculum are analyzed and how they adapt the official curriculum and thus make contributions to improve the Colombian intended mathematics curriculum and even serve as a basis for comparison the institutional curriculum, the lesson plan (or classroom) and the implemented curriculum of a different sample of schools. Future research could also investigate: what mathematical connections about exponential and logarithmic functions are promoted in textbooks used by teachers in the classroom? What mathematical connections about exponential and logarithmic functions are promoted in the classroom by the teachers and what are evidenced by students? In addition to proposing designs on exponential and logarithmic functions that promote making mathematical connections and therefore impact the development of student understanding.

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