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The Influence of Infusion Learning Strategy on Students' Mathematical Argumentatıon Skill

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This study aims to see the influence of infusion learning strategy on students' mathematical argumentation, in particular to those who were prospective math teachers. Method: It used an experimental research design involving 70 respondents. The experimental group implemented an infusion learning strategy, while the control group applied the conventional strategy. Both experimental and control groups had post-tests for data collection dealing with students' mathematical argumentation. The data would further be analyzed by using t-test through SPSS software. Findings: The result shows a significant difference between those two groups. Therefore, this study defines that infusion learning strategy brought effects on the mathematical argumentation of prospective math teachers. Implications for Research and Practice: The limitation of this study is the infusion learning strategy has only been tested on small samples. The researcher's suggestion is that further research can be implemented the infusion learning strategy to a larger sample. The mean score of post-test in the experimental group is 77.8571, while the mean score of post-test in the control group is 56.3143. It indicates that the mean score of experimental groups is significantly higher than the control group, and thus, it shows a significant difference between them on their scores.

Keywords: proof, mathematical argumentation skill, pre-service teacher, infusion learning, learning strategy

INTRODUCTION

When solving an proofing problem, problem solvers require argument support (Krummheuer, 1999; Cho, & Jonassen, 2002; Hoyles, & Küchemann, 2002; Verheij, 2005). This argument is needed to determine, produce and support a reasonable solution. Through argumentation, problem solvers can give reasons to strengthen or oppose, support or reject an idea. When a problem solver has the ability to argue, he or

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she can justify his or her solutions and actions, so he or she can leave doubts and uncertainty in solving a problem. He or she also feel free in choosing it. In addition, he or she can choose rational solutions.

One problem of teaching mathematics at the university level is students' low competence in mathematical argumentation, especially in solving argumentation problems. In the context of mathematical argumentation, some students sometimes do not use deductive argument (Inglis, Mejia-Ramos, & Simpson, 2007; Tristanti, 2019). Whereas, a deductive argument is the only argument considered as a valid argument, since its premises are based on verified definition, theorem, and/or facts (Rodd, 2000; Harel, 2001; Tall, 2004; Lodder, 2009). Researchers conducted preliminary research to examine students' mathematical argumentation skills. The initial research was conducted on 5 - 26 April 2021 involving 30 participants. The results of the initial study showed 30% of students could not express arguments, 40% of students could express some statements but could not argue, 30% of students could express arguments but be incomplete. It indicated that the students' mathematical argumentation ability was low. The difficulty of students in building arguments was due to the lack of pedagogical skills for teachers to develop argumentation skills in the classroom. Students were usually asked to solve proof problems directly without going through an argumentation process.

There are students who do not use deductive arguments, where students use intuitive and structural intuitive arguments (Inglis et al, 2007). An inductive argument is when students make sure for themselves and persuade others about the truth of the allegation by evaluating the allegation in one or more specific cases to reduce the uncertainty of a conclusion. The intuitive structural argument is when students use observation about, or experiment with, a kind of mental structure, be it visual or vice versa, which persuades them to conclude.

In building deductive arguments, the first students use non-deductive arguments. There are students who start from an intuitive structural argument, then they can build a deductive argument (Tristanti, et al 2015). There are students who start from inductive arguments then they can build deductive arguments (Tristanti, et al 2016). Students also get an argument scheme malformation in the construction of evidence (Fuat, 2020). Schematic arguments reach malformation in 0,1,2,3 and 4. Malformation 0, namely students cannot express arguments. Malformation 1: students can express some pronouncements but not arguments. Malformation 2, namely students can express some pronouncements but their argument is incomplete. Malformation 3: students express arguments without conclusion. Malformation 4: students express incomplete arguments.

Students' low competence in mathematical argumentation is due to their less understanding of the importance of argumentation. Furthermore, they are not trained to carry out good argumentation. To develop their argumentation competence, non-deductive argumentation should be shifted into the deductive argument (Harel, & Sowder, 1998). Therefore, the teaching objectives should be clearly explicit in order to improve students' mathematical argumentation competence to reveal formal evidence gradually (Harel, 2001). Finally, students judge the difficulty in understanding and evaluating mathematical arguments because of their difficulty in understanding and using natural mathematical statements to prove it.

Many researchers that apply learning models or strategies to develop students' mathematical argument skills have been carried out, such as Delen (2017), Liliarti & Kuswanto (2018), and Lin (2018). The result of Delen's research (2017) shows that the use of Facebook groups can be an alternative in teaching argumentation. The results of Liliarti & Kuswanto's research (2018) show that Android-based learning media with local content in the form of othok-othok toy boats can increase the competence of diagrammatic and argumentative representations in physics learning. The media can be used as a medium for learning physics both inside and outside the classroom. While the result of Lin's research (2018) shows that developing students' argumentation via conjecturing with scaffolding or questioning by the teacher in a classroom.

It appears that previous research has not found the learning strategies used to improve mathematical argumentation skills at the university level. The ability of mathematical argumentation can be developed in mathematics learning through the application of infusion learning strategies. Infusion Learning Strategy is a learning strategy that aims to assist students in developing their competence in mathematical argumentation. The infusion learning strategy phases are actively thinking, having argumentation out of dialogue, having argumentation in a small dialogue, and having argumentation in class dialogue.

The infusion learning strategy for teaching mathematical arguments is defined as combining several skills during the teaching and learning process. In this strategy, mathematical argumentation skills are taught and identified along with learning content as implemented by Davies (2006); Noor & Sazeli, (2010). This infusion learning strategy refers to the infusion learning approach which is first introduced by Swartz (1987). The infusion learning approach aims to teach certain critical thinking skills along with different subjects, and inculcate critical thinking skills through the teaching of defined learning materials. Kurniati et al (2020) apply infusion learning during 10 meetings in the Real Analysis course with sequential properties of real numbers and algebraic nature. The finding is that after participating in infusion learning, students get changes in truth-seeking behavior. This change is a habit that participants always do in response to a given mathematical problem.

This study aims to determine the effect of learning with the Infusion Learning Strategy in order to improve students' competence in mathematical argumentation. As applying the strategy, three advantages are expected, as follow: (1) resulting in a product of Infusion Learning Strategy, (2) improving students' competence on mathematical argumentation, and (3) automatically strengthening their competence in solving problems of proving which is the core of learning mathematics in university level.

Context and Review of Literature

Mathematical argumentation

The terms argument and argumentation reflect two definitions. The term argument refers to product and argumentation refers to the process (Kuhn, & Udell, 2003). Someone constructs an argument to support a claim. On the other hand, a process of dialogue through which two or more people are engaged in debating the claim is called argumentation of argumentative discourse. Argumentation is a kind of someone's rhetoric to influence other's arguments and attitudes just so they believe and finally behave as what the author or speaker expected (Keraf, 2010). From such description, it is clearly apparent that, through argumentation, someone tries to construct some facts for showing whether or not an argument is right. The facts he or she uses should be reasonable, not just because of his or her preference or emotional approach, so that he or she may deliver his or her argument along with its reasonable evidence respectively and critically.

The mathematical argument is a dynamic process of social discourse to find new mathematical ideas and convince others that the claim is true (Rumsey & Langrall, 2016). Justification is part of a mathematical argument because students provide evidence and reasons to convince others that their claims are valid. Meanwhile, according to Inglis, Mejia-Ramos, & Simpson (2007), mathematical arguments include informal reasoning and formal evidence. Arguments can be seen both as elements and as products of mathematical reasoning processes (Viholainen, 2011). The purpose of the reasoning process is to build an argument. This process can be inductive, deductive, or the use of intuition in making and testing guesses. Based on the opinion of Rumsey & Langrall; Inglis, Mejia-Ramos, & Simpson, and Viholainen that mathematical proofs and reasoning are types of mathematical arguments.

In a logical perspective, a valid argument is an argument that is based on an acceptable premise and uses rules in concluding, so as to produce an acceptable conclusion (Lodder, 2009). Deductive arguments only fulfill valid arguments because the premise is based on definitions, theorems, or facts that have been verified. A valid argument if the form is valid (Purwanto, 2015). The purpose of a valid form of argument is that each premise is properly substituted with any particular statement producing a correct conclusion.

Formally valid arguments are arguments based on deductive thinking (Toulmin, 2003). Whereas a valid argument is not formally an argument that is based on non-deductive thinking. Valid arguments are arguments based on correct interpretation (Kane, 1990). While Nussbaum (2011) reveals that a valid argument is an argument that can be accepted by others without a rebuttal. Kane and Nusbaumm's opinion state that the validity of an argument is based on a rhetorical perspective. Arguer concocts facts, so he can show and convince others about an opinion that is true or not. A valid argument in the rhetorical perspective is not only from the correct premises and procedure but the conclusion can be accepted without rebuttal (Lodder, 2009).

From some of these opinions, it can be concluded that the mathematical argument in this study is a series of mathematical statements consisting of hypotheses (or premises), and conclusions. Valid arguments are arguments based on true and correct deductive thinking. Deductive thinking can be seen when the arguer uses definitions, axioms, rules, algebraic manipulation, or the use of examples of denial in his or her mathematical arguments. The conclusions generated by the arguer can also be accepted logically.

Statements on mathematical argumentation are seen as a kind of argumentation which structure corresponds to what has been developed by Toulmin. This model is used to compare and analyze the content of argumentation and proving from a cognitive perspective. It is known as Toulmin Scheme (Toulmin, 2003). The scheme consists of three components, such as claim (C), in the form of speaker's statement/utterance, data (D), in the form of data justifying the claim (C), warrant (W), in the form of rules of inference which makes data (D) connect to the claim (C) and backing/support (B). In argumentation, the first stage is a statement/premise based on the perspective of someone who is arguing things. This stage is called claim (C). The second stage is data exploring (D) which aims to support the claim. To correlate between C and D, warrant (W) presents to do justification on the data to make it easy to understand by showing the correlation of those two components (i.e., C and D). In case that there are rules (W) that are not yet revealed, other pertinent rules (i.e., Backing) can be taken into account. The following figure presents Toulmin Scheme.

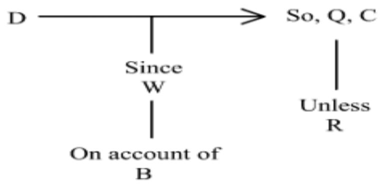

Figure 1
Toulmin Scheme

There are two types of arguments namely formal and informal arguments (Viholainen, 2011). The formal argument, when the warrant is based on definitions, axioms, and theorems. Generally, formal arguments are more thorough and detailed, so that formal arguments can be used to remove all doubts and uncertainties from the truth of a statement. Whereas informal arguments, when the warrant is based on concrete interpretations of mathematical concepts, are based on visual or other illustrative representations. The characteristics of informal arguments are that mathematical concepts are interpreted by using illustrative representations, for example, mathematical concepts can be illustrated by several physical contexts. The representation depends on personal experience, situational factors, and the field of mathematics.

Infusion learning strategy

Before someone constructs an argument, he or she should have logical reasoning at first (Walton, 1990). Mathematical reasoning is the main component of thinking that involves the construction of generalization and figures out a valid conclusion about ideas and those ideas are interrelated to each other (Artzt, & Yaloz-Femia, 1999; Peressini, & Webb, 1999; Krulik, Rudnick, & Milou, 2003). The example of having logical reasoning which is not in the form of argument is when playing chest. It is about having logical reasoning when understanding an explanation. Furthermore, another example that shows logical reasoning with the argument is when someone speaks perfectly and it is easy to understand by others. It should be noted that logical reasoning differs from having argumentation. Logical reasoning may happen without any specific purposes/aims while having argumentation aims to reveal a directed argument that convinces and ensure others to receive and understand any explanation an arguer has just delivered. Arguments may arise in bother dialogue and non-dialogue.

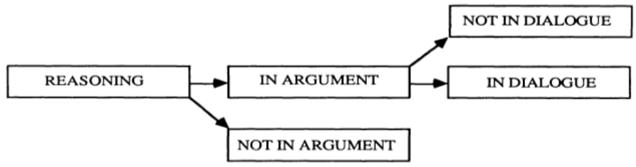

Figure 2
Correlation Between Reasoning And Argument

Such a theory by Walton (1990) applies in infusion learning strategy. It is a learning strategy that aims to assist students in developing their competence in mathematical argumentation. The infusion learning strategy phases, as follow:

Table 1
Infusion Learning Strategy Phases

Phases	Description
actively thinking	Students get a mathematical problem which asks them to investigate the truth of a statement. They are asked to think actively to construct ideas and apply them for argumentation of the problem
having argumentation out of dialogue	Students are asked to show and convince the right view through an argument which is referred to them. They try to convince themselves, and thus, they have an approach and self-debate.
having argumentation in a small dialogue	Students are divided into small groups consisting of 3 students for each. The division is based on the heterogeneity of ideas in solving problem of argumentation. They have to discuss critically in which each of the member try to express their correct view through an argument they refers to another member. Having argumentation in a dialogue aims to make their speaking skill perfect and easy to understand, as well as having other's acceptance which make them be sure and believe to what the speaker says
having argumentation in a class dialogue	A student expresses his or her arguments in his or her class and the other students responded to his argument. This phase aimed to make his or her speaking skill perfect and easy to understand, as well as having other students' acceptance since they are sure and believe to what he or she says.

METHOD

Experimental Design

This study was a quantitative experimental method since this study aimed to ensure the effect of treatment of learning with Infusion Learning Strategy on students' mathematical argumentation. While the design of this research was a posttest-only control group design. The following figure 3 presented the research design of this study.

E	X	O1
C		O2

Figure 3
Experimental Design

E was the experimental group, while C was the control group, and X was the treatment to the experimental group, such as a learning implementation by Infusion Learning Strategy. O1 was the post-test result of students in the experimental group, particularly to their mathematical argumentation after having such treatment. O2 is the the post-test result of students in control group; those with no treatment.

The treatment was considered having significant effect on students' competence in mathematical argumentation if it found a significant difference in the result of post-test between experimental and control groups. Although the control group was not given the same treatment as the experimental group, it did not mean that the control group had no treatment or teaching. The control group would have an instruction but it was not Infusion Learning Strategy. The following hypotheses of this study.

H0: There was no difference in post-test score between experimental and control groups

H1: There was a difference in post-test score between experimental and control groups

Participant

The population of this study was 150 prospective-math-teacher students at a tertiary institution in East Java, Indonesia. All prospective-math-teacher students were around 20 years old. The sample of this study was 70 prospective-math-teacher students. They were divided into control group (35 students) and experimental group (35 students). The control group consisted of 10 males and 25 females, while the experimental group consisted of 11 males and 24 females. The sample was chosen by using random techniques, it randomly selected two groups; experimental and control groups, i.e the sample class was chosen randomly with the consideration that all classes had a homogeneous average of mathematical abilities. This was based on the preliminary test result data given to all classes. So the control group and the experimental group had equivalent initial abilities.

Instrument

This instrument was a test. This test was a mathematical argumentation test used to collect data about students' mathematical argumentation skills. The following described the elements of students' competence in mathematical argumentation.

1. The completeness of mathematical argumentation was, as follow:
   a. Revealing facts/claims
   b. Revealing warrant
   c. Making conclusion
2. The quality of mathematical argument required students to use deductive argument correctly.

The data of students' mathematical argumentation competence was analyzed quantitatively by giving scores to each of the elements. The guidelines of scoring were, as follow:

1. Score 2, If the students revealed the elements correctly
2. Score 1, if the students revealed the elements wrongly
3. Score 0, if the students did not reveal any element

This test was in the form of a description and consisted of 2 evidentiary problems. Before the test was given to students, it was validated by experts, namely 2 (two) mathematics education experts, and 1 (one) mathematician. The validation was carried out to meet the content and construction validation criteria. Validation criteria were identified in three components, namely the suitability of the content of the material, the construction of the material, and the suitability of the language.
After the expert declared valid, this test was tested for validity and reliability. Validity test used product moment correlation test r_{xy} through SPSS. The number of questions was considered valid if the r_{xy} value was at least 0.40. While the reliability was used Cronbach's Alpha r_{11} through SPSS. The test was considered reliable if $r_{11} > 0.60$ (Arifin, 2010). So this test was validated and item-analyzed before being given to the experimental and control groups.

Procedures

Learning with Infusion Learning Strategy was implemented in experimental group for three meetings and posttest was given at the fourth meeting. It took 100 minutes for each meeting. In control group, the students got the same material as the experimental group had. However, they did not apply Infusion Learning Strategy as the treatment. Furthermore, both groups had the same validated post-test which was the test mathematical argumentation.

Technique of Data Analysis

The data of this study was in the form of students' post-test scores to be analyzed by using a statistic calculation. To see the treatment (i.e., the effect of learning with Infusion Learning Strategy) on its significance, a difference-test analysis using t-test was conducted. The treatment was considered having significant effect if it found a significant difference between experimental and control groups on their post-test score.

The criteria of supporting H0 was "H0 was supported if the Sig. value ≥ 0,05", indicated not difference between experimental and control groups on their post-test scores. If not significant difference was found between those two groups, it indicated that the treatment gave not significant effect on the experimental group. In this case, the treatment referred to the implementation of an Infusion Learning Strategy in a learning process.

Before analyzing the data through t-test, a test of normality and homogeneity should be conducted at first. The test of normality aimed to see whether or not the data distribution was normal, the data distribution was considered normal if the valued of Asymp. Sig. was > 0.05. The test of homogenity aimed to test the similarity between both groups; experimental and control groups. This test aimed to compare two groups of data, whether or not they had similarity in variance. Both of the groups could be compared only if they had the same variance. The data were considered homogeneous if the Sig. value > significant rate at 5%.

FINDINGS

Instrument Development

The research instrument is post-test for the experimental and control groups. The test is used to describe students' mathematical argumentation skills. This test is about proof problems, here is an example of proof problems on tests:

Prove the following statement!

1. The base and top planes of a truncated pyramid are congruent
2. Volume of the cut pyramid $= \frac{1}{3}\, t\,(A^2 + B^2 + AB)$, t is the height of the cut pyramid, A is the base area of the cut pyramid and B is the top area of the cut pyramid

Figure 4
Test instrument

Before the test is given to students, the test is validated by experts. Based on the expert validation test, the instrument developed by the researchers is declared feasible to use. However, there are some revisions, namely: the use of equations in writing mathematical symbols; the command sentence is shortened and clarified; alternative answers should use 3 (three) columns containing no, statements, and reasons and it should be presented in the Toulmin scheme. Therefore, the instrument is revised according to the suggestions submitted by the validator.

After the validator states that the instrument meets the valid criteria, the instrument is distributed to certain classes for validation and reliability purposes. The particular class is selected as the place to test the validity and reliability of the post-test. Then, the test score is analyzed on its validity by using product moment correlation test r_{xy} through SPSS. The number of questions is considered valid if the value of r_{xy} is at least 0.40. for reliability, it uses Cronbach's Alpha r_{11} through SPSS. The instrument is considered reliable if $r_{11} > 0.60$. The result of validation test through SPSS is presented in the following table.

Table 2
The Result of Instrument Validation Test

	Correlations	Sig. (2-tailed)
Problem 1	.709	.000
Problem 2	.770	.000

Table 2 shows that the correlation of both problems 1 and 2 respectively are 0.709 and 0.770. It indicates that both problems are valid, as they are greater than 0.40. Furthermore, it has been found that the value of Cronbach's Alpha r_{11} is 0.782, indicates that the instrument is reliable.

The following are the results of descriptive statistics from students' argumentation ability data.

Table 3
Descriptive statistics

	N	Minimum	Maximum	Mean	Std. Deviation
Control	35	35.00	75.00	56.3143	11.47061
Experiment	35	70.00	90.00	77.8571	5.97614

The mean score of post-test in the experimental group is 77.8571, while the control group is 56.3143. The following are the results of the normality test, homogeneity test, and t-test.

Test of Normality

The following table presents the result of normality test on post-test data in experimental and control groups.

Table 4
Result of Normality Tes

	posttest_control	posttest_experiment
Chi-Square	5.971[a]	5.714[b]
Df	5	4
Asymp. Sig.	.309	.222

The output of SPSS shows that the value of Asymp. Sig for experimental group on their post-test data is .222 > 0.05, while the control group is .309 > 0.05. It indicates that the data distribution of post-test in both experimental and control groups is normal.

Test of Homogeneity

The following table presents the result of the homogeneity test.

Table 5
Result of homogeneity test

Levene Statistic	df1	df2	Sig.
13.326	1	68	.001

The result of the homogeneity test as presented in Table 5 shows that the Sig. value is .001 < 0.05. It indicates that there is no homogeneity in the data of both experimental and control groups. As the data of both groups are not homogeneous, it uses equal variances not assumed t-test.

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The following table shows that the result of equal variances is not assumed as t-test through SPSS to test the hypotheses of this study.

Table 6
Result of t-test

t-test for Equality of Means	T	-9.854	-9.854
	Df	68	51.191
	Sig. (2-tailed)	.000	.000

Based on Table 6, the Sig. value (2-tailed) is .000 < 0.05. Thus, H0 is not supported, while H1 is supported since a difference in post-test score between experimental and control groups was found. Thus, the implementation of learning with the Infusion Learning Strategy affects the competence of mathematical argumentation students. Supported this finding, the following table presents the mean score of post-test between those two groups.

Table 7
Mean score of post-test

	Group	N	Mean	Std. Deviation	Std. Error Mean
Post test score	control group	35	56.3143	11.47061	1.93889
	experiment group	35	77.8571	5.97614	1.01015

The mean score of post-test in the control group is 56.3143, while the experimental group is 77.8571. It indicates that the mean score of experimental groups is significantly higher than the control group, and thus, it shows that there is a significant difference between them on their scores.

DISCUSSION

The results show that there is a significant difference between the posttest score in the experimental group and the posttest score in the control group (students' mathematical argumentation competence). Students in the experimental group have a higher posttest score than students in the control group. The experimental group is a class that is given treatment in the form of the implementation of learning with the infusion learning strategy. This means that the infusion learning strategy can be used to train the competence of mathematical argumentation of prospective mathematics teacher students, and it is the main finding of this study.

Based on the search of researchers in previous studies, researchers have not found a learning strategy that is used to improve or practice the ability of mathematical argumentation at the university level. Previous studies focus on analyzing students' mathematical argumentation, not on learning strategies that can train students' mathematical argumentation. As research conducted by the pattern of arguments and dimensions of scientific practice of high school students when discussing (Jiménez-Aleixandre, Muñoz, & Cuadrado, 2000), activities to develop arguments and evidence under the guidance of teachers (Durand-Guerrier et al, 2011), methods of analyzing the process of interaction in mathematics classrooms - analysis of arguments and analysis of participates (Krummheuer, 2015), building deductive arguments through inductive arguments (Tristanti et al, 2016), types of mathematical arguments (Tristanti et al, 2017), and the thought process in building arguments (Tristanti, 2019), a description of the causes of student perplexity that describes the initial process of reflective thinking through arguments when students respond to a problem (Hidajat, & Sa'dijah, 2019), explores the concepts of argumentation, reasoning, and proof as understood by mathematicians and educators and presents some of the implications for mathematics education (Hanna, 2020).

These researches are focused on analyzing students' mathematical argumentation, not on learning strategies that can train students' mathematical argumentation in proofing. But it does not mean that these studies are not important. These studies are very important because all of these results of research are the basis of researchers to found infusion learning strategy that can train students' mathematical argumentation to find out the mathematical argumentation of students.

The result of this study finds that implementing an infusion learning strategy can improve students' mathematical argumentation competence. It also finds that students' competence in argumentation is progressed/improved. Durand-Guerrier et al, (2011), Boero (1999), and Boero et al (2010) also suggest that having logical reasoning during an argumentation plays an important role to construct arguments. This allows students to intentionally explore a variety of alternative ways to define a statement (notion) and justify whether or not the subsequent notions will be logical. Any correlation between argumentation and proof in mathematics can be considered as reasonable justification (Pedemonte, 2007).

The implementation of infusion learning strategy may increase the number of students in revealing facts/claims (D), warrants, and conclusions as well. Hence, they enable to identify what becomes facts in argumentation. It is the initial asset for students to prove by applying mathematical and logical argumentation rules. They enable to identify what becomes the conclusion in mathematical argumentation. Such a conclusion is considered as the final phase in argumentation, and it constitutes the result of the argumentation process. Students enable to show warrants as things that bridged facts, arguments, and conclusions. In this case, the rules applied are officially mathematical theorem and axiom.

The implementation of infusion learning strategy may also improve students' competence in mathematical argumentation and proving. Both argumentation and proof in mathematics develop when someone wants to ensure himself or others about the truth of a statement (Hanna, 1989). Therefore, this study can be a foothold to implement a learning strategy that improves students' competence in mathematical argumentation. However, it still needs further researches to see the effectiveness and influence of infusion learning strategy in improving students' competence on mathematical argumentation and proving with a bigger sample.

Before implementing an infusion learning strategy, the lecturer must ensure that students have initial abilities that are basic material and prove. This initial ability is used by students in producing mathematical arguments. As a result of initial abilities that are not owned by students is that students do not know how to start building evidence. Alcock & Weber (2010) states that the inability to use definitions in formal mathematics and poor understanding of important mathematical concepts is one of the causes of students' difficulties in constructing evidence. A common mistake in writing evidence is that students don't know how to start writing it (Stavrou, 2014).

The results of this study are in accordance with the research results of Tristanti and Nusantara (2021) that the application of infusion learning strategies can develop students' mathematical argumentation skills. The increase of argumentation skills occurs because of the infusion learning strategy as a means of training students' argumentation skills. Argumentation skills are determined by the quantity of practice. The more students practice, the more skilled the argument will be. Osborne (2005) states that arguing is a long process that requires repeated experience and practice. In addition, the increase of argumentation skills occurs because the infusion learning strategy provides opportunities for students in terms of a strong understanding of factual, conceptual, and procedural knowledge, demonstrates effective and accurate communication skills both orally and in writing, works closely, and disciplines in small dialogues.

The last discussion is about the causes of the low control group's post-test results. If seen from the mean of the control group’s post-test which are quite low compared to the experimental group’s post-test, this can be due to the material discussed is about geometry which students have already received so that for the students in the control group is rather than difficult in answering questions because in the learning process they are not specifically trained or accustomed to the dealing with proof problems. Students in the control group are not specifically trained to express statements in the form of mathematical arguments, nor are they trained in preparing valid mathematical arguments to convince others in a dialogue. In the control group, the lecturer provides proof of problems and immediately asks students to discuss in solving them. So students are not ready to solve problems and discussion. The limitation of this study is infusion learning strategy is only tested on small samples. The researchers’ suggestion for the next research is that this learning strategy can be implemented on a larger sample.

CONCLUSION

This study can prove that the infusion learning strategy in the learning process can affect the mathematical argumentation ability of prospective mathematics teachers. The phases of infusion learning strategy involve actively thinking, having argumentation out of a dialogue, and having argumentation in class dialogue. To train students' competence in mathematical argumentation, the researchers suggest to implement an infusion learning strategy in the mathematical learning process. Before implementing the infusion learning strategy, the lecturer should consider students' understanding of proofs and mathematical concepts. Suggestion for the next research is that this learning strategy can be implemented on a larger sample and consider the group of prior knowledge of mathematics (top, middle, and bottom). The specific implications of this study for research and practical context are that: (1) the result of this study may inspire education observers, especially those who deal with mathematical argumentation to do further research, and (2) infusion learning strategy can be useful as an alternative to explore students’ mathematical argumentation competence in mathematics class, and as the result, it may bring positive effects to their problem-solving skills, in particular to argumentation.

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