International Journal of Instruction *January 2025 ● Vol.18, No.1*
e-ISSN: 1308-1470 ● www.e-iji.net *p-ISSN: 1694-609X*
pp. 325-340

Article submission code: 20240425184253
Received: 25/04/2024
Revision: 08/08/2024
Accepted: 19/08/2024
OnlineFirst: 03/10/2024

Examining Procedural Proficiency in Fraction Addition: Comparative Insights from England and Taiwan

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In recent decades, the teaching and learning of fractions have remained challenging for both teachers and students. This study reviewed fraction curricula in England and Taiwan, revealing a significant disparity that drives an investigation into how these different curricula impact students' procedural proficiency in fraction addition in these two regions. A comparative analysis of 561 British and 648 Taiwanese students, aged 12 and 13, was conducted using a written test to assess variations in procedural proficiency in fraction addition and explore the extent of these differences. Through thorough coding of students' strategies employed during the test, the study identifies the persistent impact of whole number bias on procedural proficiency in fraction addition, especially in England. Results show significant differences in proficiency between regions, with Taiwanese students demonstrating greater success in algorithmic application. No significant age-related differences were found within the same educational system, suggesting that mastery of fraction addition does not necessarily improve with age or increased exposure. These findings not only emphasise the need for targeted improvements within educational systems but also underline the importance of further international research to uncover and identify the diverse factors influencing mathematical achievement across different regions.

Keywords: procedural proficiency, fractions, fraction addition, comparative study, age and procedural proficiency, England, Taiwan

INTRODUCTION

The teaching and learning of fractions remain challenging for both teachers and students (Azid et al., 2020; Schulz, 2023). This phenomenon is concerning because, from a practical standpoint, fractions play an important role in dealing with quantitative information in everyday situations, such as measuring ingredients for recipes, comparing prices, or calculating speed and distance travelled. Moreover, from a mathematical perspective, lacking successful development of fractions in the early years of children's learning will result in difficulties that persist into secondary years of schooling (Nickson, 2000). The Department for Education in England (2011, p. 72) emphasised that 'proficiency in fractions is considered essential for accessing the

Citation: Li, H. C. (2025). Examining procedural proficiency in fraction addition: Comparative insights from England and Taiwan. *International Journal of Instruction*, *18*(1), 325-340.

secondary mathematics curriculum, in particular in the domains of measure, algebra and geometry as well as probability'.

Research has argued that students sporadically utilised numerous erroneous rules, and individual differences in the application of various strategies and procedural skills varied more among students than anticipated (Alabdulaziz, 2024). Over the past two decades, many questions about "procedural proficiency"—a term used in this study to encompass both procedural fluency and conceptual understanding of operations—remain unanswered. This research gap motivates the present study. Specifically, research on this issue will greatly benefit from cross-cultural, comparative studies, where different educational systems serve as natural laboratories to uncover cultural practices and identify variables stemming from fundamental assumptions about mathematics teaching and learning (Stevenson & Stigler, 1992).

In mathematics education, influential international studies like Trends in International Mathematics and Science Study (TIMSS) and Programme for International Student Assessment (PISA) have assessed students' mathematics achievements on a large scale since the 1950s. The results of TIMSS and PISA consistently demonstrated that Taiwanese students outperformed British students (Schleicher, 2023). However, there has been limited research investigating students' mathematical performances between England and Taiwan. Specifically, cross-cultural comparisons of procedural proficiency in fraction addition have received insufficient attention, particularly in exploring students' struggles in this specific area. To address this gap, this study conducted a small-scale cross-cultural, comparative analysis to explore procedural proficiency in fraction addition among British and Taiwanese students.

Inhibition of the Whole Number Bias in Fraction Addition

What makes the teaching and learning of fractions so hard? In practice, a conventional concrete approach to learning fractions involves thinking in terms of partitioning or equal sharing. This approach is typically based on discrete, countable objects, as well as objects requiring the dissection of a continuous whole. However, this concrete approach alone does not fully convey the concept of fractions. For instance, the notion of "equal sharing" is just one of many properties of fractions and is insufficient to convey the concept meaningfully to students. Tsai and Li (2017) argued that typical fraction instruction in schools overemphasizes a part-whole construct, leading to an understanding of 3/5 as three out of five parts rather than as the iteration of three abstract units of size 1/5.

While there may be many reasons why fractions are challenging to grasp, much research has identified the "whole number bias" as a major source of difficulties with students' understanding of fractions. The whole number bias refers to the tendency to apply characteristics of whole numbers in tasks involving fractions, even when such application is inappropriate or incorrect. (Van Hoof et al., 2020). Such misunderstanding contributes to difficulties in dealing with various problems, including ordering, comparing, and identifying equivalent fractions. For instance, students might mistakenly believe that 3/8 is greater than 3/5 because 8 is larger than 5, or that 2/4 is greater than 1/2 because 2 is larger than 1.

Furthermore, the process of adding fractions necessitates the conversion of fractions into equivalent forms with the same denominator, which may contradict students' existing understanding of whole numbers. From a young age, children develop counting skills and use them to solve practical arithmetic problems. However, transitioning from counting whole numbers to fractions can pose challenges, as whole number concepts may hinder the development of fraction understanding (Namkung, et al. 2018). Students' difficulty with fraction addition may stem from their tendency to apply whole number properties to fractions, viewing fractions as collections of separate whole numbers. For instance, they may see 2/3 as two and three separately rather than as a single entity, leading to challenges in recognising 3/5 as the result of 3 ÷ 5.

Understanding fractions requires a departure from basic principles of whole number concepts, as children need to construct a new understanding of fractions. However, even after this conceptual change occurs, the initial whole number representation may continue to persist and can influence computational skills involving fractions. Consequently, adults may still exhibit the whole number bias in fraction tasks, where whole number representations interfere with the accurate understanding of fractions. This phenomenon emphasises the complexity of transitioning from whole numbers to fractions and highlights the importance of addressing and overcoming the whole number bias in mathematics education (Obsteiner et al., 2013).

Fraction Programmes in England and Taiwan

Formal schooling begins at age five in England and at age six in Taiwan. Consequently, a Taiwanese Grade 1 student is equivalent to an British Year 2 student in terms of age. Therefore, this study uses "Grade 1", "Grade 2", and so on to denote the grade year of Taiwanese students, and "Year 1", "Year 2", and so on for the school year of British students.

In England, the introduction of fractions starts with unit fractions such as 1/2, 1/3 and 1/4 in Year 3. In Year 4, students are expected to have the ability to recognise simple fractions and mixed numbers and begin to relate fractions to division and use ideas of simple ratio and proportion (the quotient and ratio constructs). The concept of an improper fraction and how to change an improper fraction to a mixed number are introduced in Year 5. Students also learn to add and subtract fractions with the same denominator and denominators that are multiples of the same number. By Year 6, how to reduce a fraction to its simplest form by cancelling common factors in the numerator and denominator is introduced. In Year 7 and Year 8, the first two years of secondary schools, students begin to add and subtract simple fractions and those with common denominators; multiply and divide an integer by a fraction and calculate simple fractions of quantities and measurements. Year 9 uses efficient methods to add, subtract, multiply and divide fractions.

In contrast, the curriculum in Taiwan appears more intensive. By Grade 3, students are expected to recognise fractions (not necessarily restricted to proper fractions) and add and subtract and compare simple fractions and those with common denominators. In Grade 4, recognising simple fractions, mixed numbers and improper fractions and comparing fractions with different denominators are introduced. By the end of Grade 5, students learn all the concepts and skills of fractions. In Grade 6, students are expected to be more proficient in computations and problem-solving. After Grade 6, the Taiwanese curriculum does not include specific teaching objectives related to fractions. It appears that in England, the curriculum adopts a spiral approach, revisiting and further developing each subject area across primary and secondary school years, while the curriculum in Taiwan typically covers subject areas once at appropriate grade levels, with limited repetition. Moreover, teaching procedural knowledge differs significantly between England and Taiwan, with Taiwan emphasising computational fluency much more in its curricula.

Furthermore, instruction about the concepts and skills of fractions are divided into two steps in the England's curriculum: introduction at primary schools and further development at secondary schools. In contrast, the Taiwan's curriculum integrates instruction on fractions throughout students' primary education, with an emphasis on proficiency in fraction calculations. British students typically learn concepts before calculations, contrasting with Taiwanese students' experiences. This disparity drives the study to investigate how different curricula impact students' development of fraction addition in England and Taiwan.

Aims and Research Questions

In mathematics education, fractions have long been a challenging topic for students to grasp. For many school children, mastering the arithmetic of fractions is often a matter of rote memorisation in mathematics classes, leaving their procedural knowledge of fractions fragmented rather than interconnected. This cross-cultural, comparative study aimed to uncover underlying procedural misconceptions and challenges that students have regarding fractions. It examined the strategies employed by students when adding fractions, thus shedding light on the challenges they encountered. Comparing students' performance with that of others helps the present study understand their relative success or failure. It is important to note that the aim of the study is not to establish superiority of one country over another but to provide insights into areas of success and areas needing improvement for both groups.

Since this study investigated procedural proficiency in fraction addition among students from England and Taiwan, it was crucial to recruit students who had learned fraction addition. Upon reviewing the national curriculum in both regions in the previous section, it became apparent that students in both England and Taiwan should have acquired the knowledge about fraction addition after the age of twelve. Furthermore, one common assumption regarding the relationship between age and academic performance is that it follows a linear trajectory, where older students are expected to perform better academically than younger students. Therefore, the present study also aimed to examine this assumption by concentrating on students aged 12 and 13 across two different school years in both England and Taiwan with a view to exploring the relationship between age and fraction addition. Specifically, the study intended to answer the following three research questions:

1. How does procedural proficiency in fraction addition vary between students from England and Taiwan?

2. To what extent do students in both England and Taiwan share common misunderstandings or incorrect methods when adding fractions?
3. Are there statistically differences between student age and their procedural proficiency in fraction addition within the same educational system?

METHOD

Research Design

This study was conducted in schools located in both England and Taiwan. To address the research questions, a combination of quantitative and qualitative approaches was employed. A researcher-designed written test was utilised to measure the differences in procedural proficiency of fraction addition between the two regions and explore the extent of these differences. Students' strategies employed during the written test were coded to facilitate an understanding of the meaning behind their strategies. Figure 1 illustrates the methodological steps and the important elements of each step, each of which will be described in subsequent sections.

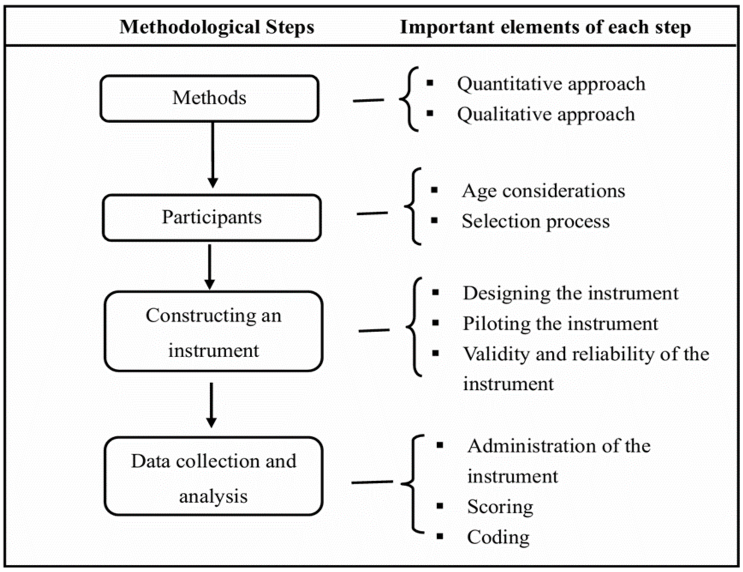

Figure 1
Methodological steps

Participants

This study specifically focused on students aged 12 and 13 in both regions to ensure that all participants had been exposed to fractional computations during their school years. Consideration was also paid to the choice between age-based and grade-based sampling strategies due to differences in starting ages and curricula. Age-based sampling aims to equalise students' maturity levels but results in disparate educational experiences. Conversely, grade-based sampling ensures similar years of formal schooling but may yield varied ages. In line with Wiley and Wolfe (1992), this study considered age-based sampling most suitable for international studies due to its potential for better international comparability.

In England, schools' performance in Mathematics, English, and Science is officially reported on an annual basis. Consequently, schools with attainment averages close to the national average were identified and approached. Schools voluntarily participated in this study. The final sample consisted of 561 British participants aged 12 and 13 from Year 7 and Year 8, comprising 289 girls and 272 boys, selected from two secondary schools in England. However, in Taiwan, there is no yearly report on schools' performance. Therefore, after contacting several schoolteachers and parents to identify schools with average performance, these schools were invited to participate in the study. The final sample of Taiwanese participants included 648 students aged 12 and 13 from Grade 6 and Grade 7, comprising 309 girls and 339 boys, drawn from two local schools in Taiwan.

Instrumentation – the Fraction Addition Skills Test (FAST)

As no test was found that was specifically concerned with the aspects of fraction addition under consideration, it was necessary to design a new instrument. Therefore, according to the curricula of England and Taiwan, and the related literature, the Fraction Addition Skills Test (FAST) was developed by the author. Decisions regarding the questions in the FAST test were threefold. First, the test was limited to proper fractions with small denominators (less than ten) to focus on exploring students' thinking, recognition, and computation with fraction addition rather than assessing diverse levels of difficulty. Second, the test duration was capped at 15 minutes to accommodate participants' time constraints. Third, given the significance of common denominator conversion in fraction addition, four levels were distinguished: (1) Level 1: adding fractions with the same denominators (2) Level 2: adding fractions where only one denominator needs conversion, (3) Level 3: adding fractions where the lowest common denominator is found by multiplying the denominators, and (4) Level 4: adding fractions where the lowest common denominator cannot be found by simple multiplication. These four levels are illustrated in Figure 2, which also includes examples of questions.

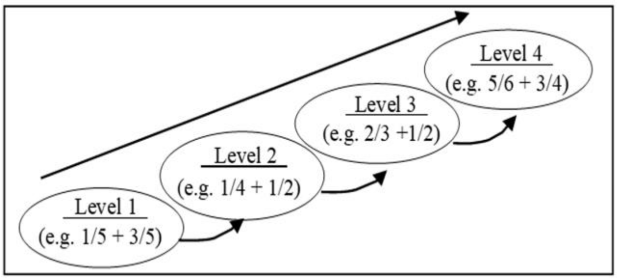

Figure 2
Four levels of difficulty in the FAST test

The items in the FAST test are identical but are presented in English in England and in Chinese in Taiwan. A pilot study of the FAST test was conducted, involving parallel groups in England and Taiwan, consisting of 22 and 35 students, respectively. Their backgrounds and ages were comparable to the participants in the formal study. These students were asked to complete a questionnaire aimed at providing feedback to enhance the quality of the FAST test. The results of this questionnaire were used to further modify items prior to formal testing.

Mathematics educators in both England and Taiwan were invited to review all the items in the test to ensure the content validity. To check whether all the questions are measuring the same underlying construct, the internal-consistency reliability of the FAST test was measured by means of Cronbach's alpha coefficient (see Table 1). Ideally, an acceptable Cronbach's alpha coefficient of a scale should be above 0.7. The Cronbach's alpha coefficients for the British group is .8243 and for the Taiwanese group, it is .8599, suggesting that the FAST test has an acceptable degree of reliability for both regions.

Table 1
Cronbach's Alpha coefficient

	All (N=1209)	England (N=561)	Taiwan (N=648)
FAST test	.8840	.8243	.8599

Each test level contains only one item due to a 15-minute time constraint imposed to accommodate participants. Table 2 shows the difficulty and discrimination indices for each level item in England and Taiwan, as well as overall values. The difficulty index is calculated as the percentage of the total number of correct responses to the test item. The difficulty index is calculated as the percentage of correct responses to an item, while the discrimination index measures the difference between the percentage of correct responses in the top 27% and the bottom 27% of the group. According to Mitra et al. (2009), an item is considered difficult if its difficulty index is below 0.3 and easy if its index is above 0.8. The discriminative index classifications are: 0.0 – 0.19 (poor), 0.2 – 0.29 (acceptable), 0.3 – 0.39 (good), and >0.4 (excellent). As shown in Table 2, the test items are generally more challenging and discriminative for British students, while Taiwanese students find them easier and exhibit less discrimination. Overall, difficulty values for all students decrease from 0.81 at Level 1 to 0.57 at Level 4. In contrast, discrimination indices remain high at all levels.

Table 2
Difficulty value and discrimination coefficient

		Level 1	Level 2	Level 3	Level 4
England	Difficulty index	0.62	0.54	0.27	0.23
	Discrimination index	1	1	1	0.85
Taiwan	Difficulty index	0.97	0.89	0.88	0.87
	Discrimination index	0.11	0.40	0.44	0.48
All	Difficulty index	0.81	0.73	0.60	0.57
	Discrimination index	0.70	1	1	1

Data Analysis

Quantitative data come from test scores, while qualitative data are derived from students' strategies used in solving the questions. One point was awarded for each correct answer on the FAST test, while no points were given for incorrect answers. An inductive coding approach was utilised to identify both general and distinctive features from the strategies used by the students (Bingham, 2023). For instance, a student's strategy of "1/2 + 1/4 = 1/8" was coded as "Use multiplication principles". When coding students' strategies, no predetermined codes were established; rather, they were gradually developed during the analysis of students' strategies observed on the test papers. The coding process comprises four steps: identification, labelling, reduction, and summarisation. The inductive approaches facilitate an understanding of meaning in complex data by developing summary themes or categories derived from the raw data. SPSS version 27 for Windows was used to perform the statistical analyses. Preliminary analyses were performed to ensure that there was no violation of the assumptions of normality. Descriptive Statistics were employed to offer insights into means, standard deviations, the number, and percentages of responses, aiding the assessment of students' overall performance in the FAST test. Furthermore, Independent Samples Tests were conducted to identify significant differences in the test scores between British and Taiwanese students, as well as to examine the relationship between student age and their procedural proficiency in fraction addition.

FINDINGS

Descriptive Statistics

Descriptive statistics, outlining the characteristics of the students' performances, are presented in Table 3. It shows that in England, 62% of student responses were correct at Level 1, 54% at Level 2, 27% at Level 3, and 23% at Level 4. In contrast, Taiwanese students achieved 97% correct responses at Level 1, 89% at Level 2, 88% at Level 3, and 87% at Level 4. These results indicate that Taiwanese students performed considerably better than their British peers at all levels of the FAST test. Moreover, it also shows that the percentage of correct responses decreases as the difficulty level increases. This result supports the notion that the FAST test incorporates a hierarchy of difficulty, consisting of four levels, as described. It was also noticeable that the percentage of correct student responses decreased notably from 54% to 27% for British students as they advanced from Level 2 to Level 3. In contrast, the correct percentage only decreased by 1% (from 89% to 88%) in the Taiwanese group. Overall, the percentage of correct responses in Taiwan remains consistently above 87% across all levels, while in England, the percentage decreases from 62% to 23% as the level increases.

Table 3
Descriptive statistics of the FAST test results

Percentage of correct student responses (%)	Level 1	Level 2	Level 3	Level 4
England (N=561)	62	54	27	23
Taiwan (N=648)	97	89	88	87

Independent Samples T-Test between England and Taiwan

An Independent Samples T-Test was conducted to compare the test scores between England and Taiwan with a full score of 4 being the maximum possible (see Table 4). In England, 561 students participated, with a mean score of 1.65 and a standard deviation (SD) of 1.49. In Taiwan, 648 students participated, with a mean score of 3.59 and an SD of 1.01. The Independent Samples T-Test analysis revealed significant differences in the test scores ($t\ (1207) = -21.283$, $p < 0.0005$, effect size = 0.283) between the British and Taiwanese groups, with the latter having a higher mean score. According to Cohen (1988), the effect size can be interpreted as follows: 0.01=small effect, 0.06=moderate effect, and 0.14=large effect. Based on the results of the effect size, it shows that the magnitude of the differences in the means in the FAST test between the two groups was relatively large. These results indicate that overall, Taiwanese students performed significantly better than their British peers in the FAST test.

Table 4
Independent samples T-Test between England and Taiwan

	Number	Mean (SD) Full Score: 4	T-Test	Sig. (2-tailed)	Effective Size
England	561	1.65 (1.49)	-21.283	.0005*	0.2833
Taiwan	648	3.59 (1.01)			

Analysis of Student Strategies in the FAST Test

With the aim of identifying the students' strategies that likely led to their answers, an analysis and coding of their methods used to solve the questions, as well as the errors made when adding fractions, were conducted. 15 strategies were identified and presented in Table 5. Table 5 shows that in England, 32.7% of students used algorithms correctly (Strategy 1), compared to 89.1% in Taiwan. While 7.5% of British students created diagrams to answer correctly (Strategy 2), only 0.3% of Taiwanese students did the same. Strategy 3 (No working displayed but answered correctly) did not comply with the instruction, as during the administration of the FAST test, all students were instructed to show their calculations. The remaining strategies, from 4 to 15, were associated with the incorrect or no responses. However, Strategies 4, 6, 7, 8, 9, and 12 did not occur in the Taiwanese group. For instance, Strategy 4 occurred in 0.2% of British students, with no instances in Taiwan. This suggests that using diagrams does not necessarily lead to the correct answer, as their incomplete understanding of fractions resulted in incorrect strategies when drawing diagrams. Some errors in fraction addition were related to incorrect usage of multiplication and division algorithms, such as Strategies 10, 11, and 12, which were generally rare or absent in Taiwanese students but present to varying degrees in British students. Notably, 12.1% of British students showed no clear strategy (Strategy 13), compared to only 3.1% of Taiwanese students. Careless computational errors (Strategy 14) were more common in Taiwan (2.1%) than in England (0.9%). Furthermore, 11.1% of British students provided no responses (Strategy 14), compared to just 0.3% in Taiwan.

Table 5
Percentage of each student strategy in the FAST test

Student strategy (%)		England	Taiwan
1	Use algorithms correctly	32.7	89.1
2	Creating diagrams to answer correctly	7.5	0.3
3	No working displayed but answered correctly	0.6	0.2
4	Use diagrams in a wrong way (e.g. $\frac{2}{3}+\frac{1}{2}=\frac{3\,(parts)}{3}=1$) 	0.2	0
5	Add numerators and denominators (e.g. $\frac{1}{4}+\frac{1}{2}=\frac{2}{6}$)	31.5	4.1
6	Another type of adding numerators and denominators (e.g. $\frac{5}{6}+\frac{3}{4}=\frac{5+6}{3+4}=\frac{11}{7}$)	0.4	0
7	Add all up (e.g. $\frac{2}{3}+\frac{1}{2}=2+1+3+2=8$)	0.8	0
8	Convert into decimal in a wrong way (e.g. $\frac{5}{6}+\frac{3}{4}=5.6+3.4=9$)	0.5	0
9	Convert into decimal correctly, but answered incorrectly	0.9	0
10	Use multiplication principles (e.g. $\frac{1}{4}+\frac{1}{2}=\frac{1}{8}$)	0.6	0.6
11	Use division principles (e.g. $\frac{2}{3}+\frac{1}{2}=\frac{2}{3}\times\frac{2}{1}=\frac{4}{3}$)	0	0.2
12	Use cross-multiplication (e.g. $\frac{5}{6}+\frac{3}{4}=\frac{18}{20}=\frac{9}{10}$)	0.2	0
13	No rules	12.1	3.1
14	Careless computational errors	0.9	2.1
15	No responses	11.1	0.3

Table 6 shows the use of Strategy 2, which involves creating diagrams to answer correctly, across different levels of the FAST test for students from England and Taiwan. At Level 1, British students used this strategy 81 times, significantly more than the 2 times it was used by Taiwanese students. At Level 2, it was used 75 times in England and 3 times in Taiwan. At Level 3, there were 10 instances in England and 2 in Taiwan. At Level 4, usage in England dropped to 2, while it remained 2 in Taiwan. This data reveals a much higher frequency of Strategy 2 use among British students compared to their Taiwanese peers at all levels of the test. It also suggests that using diagrams may not be very helpful for tackling Level 3 and Level 4 questions, though they can be useful for Level 1 and Level 2. In contrast, correctly using algorithms, as shown with Strategy 1, directly led to accurate fraction addition and higher scores on the FAST test.

Table 6
Use of Strategy 2 at each level

Number of Strategy 2 usages	Level 1	Level 2	Level 3	Level 4
England	81	75	10	2
Taiwan	2	3	2	2

A significant error was adding numerators and denominators (Strategy 5), seen in 31.5% of British students and only 4.1% of Taiwanese students. It is worth taking a closer look at this strategy, and Figure 3 illustrates the percentage of using Strategy 5 in both England and Taiwan. When examining the percentage of British and Taiwanese students using Strategy 5 at each level, as shown in Figure 3, a specific trend emerges: the percentage of British students using Strategy 5 increases with the increase in difficulty level. However, this trend is not evident among the Taiwanese students, despite an increase from 2.1% to 4.4% from Level 1 to Level 2. It was also observed that for the Taiwanese group, the percentage of using Strategy 5 to add fractions remained at 4.4% even when the difficulty levels increased. This data also shows the impact of whole number bias on procedural proficiency in fraction addition, especially in England.

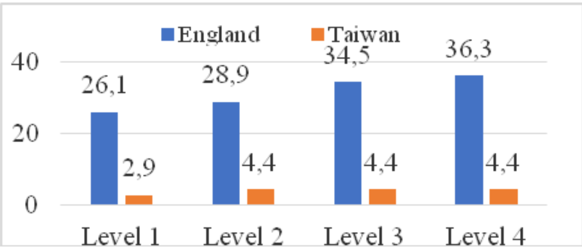

Figure 3
Percentage of strategy 5 usage

Independent Samples T-Test for Age Groups

Since the student sample of this study comprised of two age groups (12-year-olds and 13-year-olds), this offers the opportunity to examine the impact of age on mathematical progress. An Independent Samples T-Test was therefore conducted to compare the students' performance in the FAST for 12- and 13-year-olds within each region, as presented in Table 7. In England, 12-year-old students (N=268) had a mean score of 1.43 (SD=1.42), while 13-year-old students (N=293) had a mean score of 1.84 (SD=1.54). The T-Test results of the British group show that there were no significant differences in the test scores (t (559) = -2.780, p <.006, effect size=.019) with the 13-year-olds scoring higher than the 12-year-olds. In Taiwan, 12-year-old students (N=302) had a mean score of 3.58 (SD=0.97), and 13-year-old students (N=346) had a mean score of 3.59 (SD=1.06). For the Taiwanese group, similarly, the T-Test found no significant differences in the test scores (t (646) = -.193, p <.847, effect size=.000087) between the age groups, with the nearly identical mean scores between the 12- and 13-year-olds. Overall, these results suggest that, within each region, there is no significant difference in mathematical achievement between 12-year-old and 13-year-old students, as measured by the FAST test.

Table 7
Independent samples T-Test for age groups

	Age	Number	Mean (SD)	T-Test	Sig. (2-tailed)	Effective Size
England	12 years old	268	1.43 (1.42)	-2.780	.006	.019
	13 years old	293	1.84 (1.54)			
Taiwan	12 years old	302	3.58 (0.97)	-.193	.847	.000087
	13 years old	346	3.59 (1.06)			

DISCUSSION

Insights into Student Performance: Lessons learned from the FAST Test

The descriptive statistics in Table 3 and in the methods section served to theoretically validate the FAST test for assessing computational skills in fraction addition. It is important to note that since a convenience sampling strategy was used, there are limitations to the sample, and it is inappropriate to generalise the results beyond the given population. Nevertheless, comparing British and Taiwanese students' performances on the FAST test reveals that Taiwanese students have higher mean scores, indicating greater proficiency in fraction addition. The Independent Samples T-Test also shows significant differences in proficiency between the two groups. Analysis of the strategies used (see Table 5) highlights differences in approaches and challenges faced by students in each region. Taiwanese students' superior performance is partly due to a higher percentage (89.1%) proficient in using algorithms for fraction addition, compared to 32.7% of British students.

Using shaded diagrams can enhance students' understanding of fractions, which may explain why both British and Taiwanese students used Strategy 2, though more so among the British group (see Tables 5 and 6). Table 6 shows that the use of Strategy 2 declined significantly from 75 to 10 among British students from Level 2 to Level 3, providing evidence to show that diagrams may be effective for assisting students with simpler or more basic concepts (Level 1 and Level 2 questions); however, they may be less effective for addressing more complex or advanced concepts (Level 3 and Level 4 questions) if students are unable to reflect on their use of diagrams and adjust strategies required for higher-level tasks.

Integrating the use of diagrams with procedural knowledge can be challenging (Supandi et al., 2018). Students need to understand how to use diagrams as tools to support their problem-solving strategies and procedural fluency. Addressing these challenges requires an examination of whether the fractional instructions accompanied by pictorial presentations reflect a thorough and careful consideration for fostering the development of mathematical concepts and enhancing students' thinking and learning with fractions. The four difficulty levels of fraction addition (see Figure 1) identified by this study may have implications for teachers in differentiating instruction. They can provide varied levels of support and scaffolding based on students' readiness and mastery of concepts as they progress from one level to another.

Another insight gained from the comparison is that, as shown in Table 3, British students experienced a halving in the percentage of correct responses as they advanced from Level 2 to Level 3 of the test, dropping from 54% to 27%. This decline indicates a notable challenge or difficulty encountered by British students at the higher level of the test. In contrast, Taiwanese students showed a much smaller decrease of only 1% (from 89% to 88%) between the same levels. Overall, most Taiwanese students demonstrated the ability to convert fractions to equivalent fractions with a common denominator, enabling them to perform accurate and efficient fraction addition operations. However, a significantly lower percentage of British students had developed these skills. This comparison highlights differences in performance and challenges faced by students from these two groups at different levels of the test. Several factors could potentially contribute to this discrepancy. For instance, differences in educational systems, curriculum emphasis, pedagogical approaches, and learning culture may all play a role in shaping students' mathematical abilities. These findings extend beyond mere academic comparison and highlight the need for further international studies to explore the factors influencing mathematical achievement among students from different educational systems.

Age and Fraction Addition

This study focused on two different age groups (aged 12 and 13), offering the opportunity to explore the relationship between student age and their proficiency in fraction addition within the same educational system. It is commonly believed that as students grow older, they undergo cognitive development and acquire skills and knowledge that contribute to better academic performance. However, as shown in Table 7, there is no significant difference in performance between 12-year-old and 13-year-old students on the FAST test within each respective region. This suggests that, based on the results of the study, age does not appear to be a significant factor influencing proficiency in fraction addition among students within each region. It also suggests that an additional year of schooling do not necessarily guarantee automatic improvement in students' computational skills. Nevertheless, it is noteworthy that within the British group, 13-year-olds scored slightly higher than 12-year-olds, although this difference was not statistically significant.

As the National Council of Teachers of Mathematics (2014, p. 10) in the US points out, 'effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students become skilful in using procedures flexibly'. Therefore, the finding that there were no significant differences in test scores between different age groups within the same educational system suggests that the enduring influence of whole number concepts may persist and continue to affect computational skills involving fractions across various stages of development. This implies that the challenge of transitioning from whole numbers to fractions is not necessarily overcome with age or increased exposure to mathematical concepts. It also raises questions about the effectiveness of current educational practices in laying the foundation for fraction addition. This highlights the need for further research to reevaluate the methods used to promote procedural proficiency over time during formal schooling, as well as to consider alternative approaches to instruction and assessment that better address and mitigate the whole number bias from early childhood through the secondary years.

CONCLUSION

This study focused on fraction addition, specifically analysing students' performances on the FAST test. It acknowledges that the FAST test may not encompass all facets of fraction addition, and other relevant factors may not have been considered. The conclusions drawn are restricted by the methodology used and the inherent limitations of the sample size. The convenience sampling strategy limits the generalisation of this study. Future research should focus on improving the sampling strategy. Nevertheless, some valuable insights into the challenges that students in both regions faced with fractions, as well as the relationship between age and fraction addition performance within the same region, are not invalidated.

The statistically significant differences observed in the FAST test between England and Taiwan suggest that Taiwanese students exhibited greater fluency in the test compared to their British peers, a discrepancy unlikely to have arisen by random chance alone. By examining the strategies employed by students, this study provides insights into both successful approaches and challenges encountered when adding fractions. It also highlights the effectiveness of using algorithms correctly (Strategy 1) as a powerful tool in fraction addition. While the use of diagrams emerges as the second most common strategy among both British and Taiwanese students, as discussed in previous section, the finding highlights the need for enhancing students' ability to understand and utilise pictorial representations of fractions effectively.

The results of this study highlight both students' capabilities and areas needing improvement. The cross-cultural comparison of curricula between England and Taiwan highlights its impact on student achievement. The study does not seek to identify the superior curriculum but aims to uncover key predictors of student success to address challenges with fractions. Taiwan's curriculum, which emphasises algorithmic learning and computational proficiency, demonstrates the importance of providing ample learning opportunities with fraction operations. The lack of significant differences in test scores between 12-year-olds and 13-year-olds suggests that age may not strongly predict mathematical achievement within the same educational system. This reveals the persistent influence of whole number bias on procedural proficiency in fraction addition and calls for further research into the factors affecting students' procedural proficiency in fractions. It is hoped that this study's investigation will stimulate further research into exploring the underlying determinants of differences between different educational systems. Such insight can then inform educational policies and practices aimed at improving mathematical learning outcomes worldwide.

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