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THE RELATIONSHIP BETWEEN EIGHTH GRADE PRIMARY SCHOOL STUDENTS' PROPORTIONAL REASONING SKILLS AND SUCCESS IN SOLVING EQUATIONS[1]

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This study investigates the relationship between eighth grade primary school students' success levels in solving equations and proportional reasoning skills. 344 eighth grade students, who were registered at various primary schools in the central districts of Konya in the 2007-2008 educational year, participated in the research. The study had a relational survey research design. The findings indicated that eighth grade students' success in solving equations and their proportional reasoning skills were highly correlated (r=0.89). Students were most successful at finding the missing value within the proportional reasoning question types, followed by questions that required quantitative comparison and qualitative comparison.

Key Words: proportional reasoning, solving equations, primary school

INTRODUCTION

Proportional reasoning is a type of mathematical reasoning (Singh, 2000). It is defined as the ability to compare proportions and thus decide which are equivalent proportions (Baykul, 2006). Behr, Harel, Post & Lesh (1992) also defined proportional reasoning as "being able to make comparisons between entities in multiplicative terms".

Proportional reasoning is not only limited to the topic of ratios and proportions but is a skill which could and should be developed in relation to many other

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topics. Moreover, proportional reasoning is a prerequisite for the comprehension of the concept of proportion and is a more advanced cognitive skill than the comprehension of the concept of ratio (Baykul 2006; Dole 2008). Furthermore, various situations in daily life and topics in mathematics require proportional reasoning skills. Hoffer & Hoffer (1992) and Cramer & Post (1993) emphasised the importance of proportional reasoning for cognitive development and stated the critical significance of comprehending proportional states in cognitive stages. Researchers also indicated that proportional reasoning is not only central in the mathematics curriculum but also an essential indicator of higher scores in mathematics (Person et al., 2004).

Many topics in the primary school mathematics require proportional reasoning. This also supports the importance of teaching that foregrounds proportional reasoning in the topic of ratios and proportions. However, the current situation at schools suggests that this topic is not covered well and formulas and symbols are frequently pointlessly promoted (Hoffer & Hoffer, 1992). As suggested in the definition by Baykul (2006) and Dole (2008), proportional reasoning should be taught after the concept of ratios and before the concept of proportions using problems that require proportional reasoning skills and its full development should be left to time. Students may need as much as three years' worth of opportunities to reason in multiplicative situations in order to adequately develop proportional reasoning skills. Premature use of rules encourages students to apply rules without thinking and, thus, the ability to reason proportionality often does not develop (Van de Walle, 2004).

One of the prerequisites for the development of proportional reasoning is multiplicative reasoning (Vanhille & Baroody, 2002). In order to solve ratio problems students generally either use cross-product algorithm or try to solve it using aggregate reasoning because they could not learn or have forgotten other necessary operations. For most children quantitative comparisons are identified aggregately and they compare groups using either the language of additions or subtractions (Dole, 2008). Proportional reasoning has also been defined as the ability to comprehend multiplicative mathematical constructions in proportions. In algebra, this is expressed with the formula $y = mx$. Graphically, on the other hand, proportions are represented with a line going through the origin (Akkuş-Çıkla & Duatepe, 2002). Therefore, this definition precisely reveals the relationship between equations and proportional reasoning. For example, for the proportion $\frac{2}{3} = \frac{x}{9}$ (proportional representation of the equation 3x=18), using aggregate reasoning a student's response would be $x = 8$ obtained by adding 6 to 2 because 9 is also obtained by adding 6 to 3.

Some of the other concepts where the topic of ratios-proportions and in turn proportional reasoning are foregrounded are similarity, data graphics, probability and algebra. Equations are particularly important in algebra because most of us use them in order to solve problems. However, most students struggle to solve algebraic equations (Filloy & Rojano, 1989; Herscovics & Linchevski, 1994). Stacey and MacGregor (2000) reported that students frequently struggle in constructing the algebraic equation that would represent the expressions given in word problems and in learning the ways of doing symbolic operations in order to solve even basic equations. Therefore, foregrounding proportional reasoning skills in the teaching of equations could improve students' success in solving equations. For example, to solve the equation $\frac{x+2}{3}=\frac{4}{6}$, the reasoning should be like this: the denominator in the first ratio was 3, and this was duplicated to obtain the denominator in the second ratio. Thus, x+2 should be half of 4, i.e. 2. This reasoning would save the students from a certain operational load and at the same time contribute to the development of students' proportional reasoning skills. However, Weinberg (2002) stated that students were usually taught to solve this kind of equations by leaving x alone, which is accomplished by multiplicating both sides of the equation with the same value. Weinberg (2002) also asserted that course books, on the other hand, suggested using the cross-product algorithm to solve this type of proportions.

Therefore, the aim of this research was to determine to what extend proportional reasoning skills explained success levels in solving equations.

METHOD

Research Design

Survey designs aim to describe a present or past situation as it is/was and try to define the person or subject, which is the topic of the research, in its own circumstances. Among types of survey designs, relational survey is used to determine the existence or amount of joint variation among two or more variables. Relational analysis could be conducted in two ways; correlations and comparisons (Karasar, 1999). The aim of this research was to investigate the relationship between primary school students' proportional reasoning skills and their success levels in solving equations. Hence, it had a "Relational Survey Design".

Population and Sample

The population of this research consisted of eighth grade primary school students registered at various schools in the main districts of Konya, Turkey in the academic year of 2007-2008.

According to Yıldırım and Şimşek (2004), it is possible to select a totally random sample which could represent the population using statistical calculations. This is called simple random sampling. Knowledge of the characteristics of the population or the distribution of these is not a prerequisite for simple random sampling. Therefore, it is a simpler alternative which could be used when the whole list of the population exists (Sencer and Sencer, 1978; cited in Balcı, 2005).

The sample of this research consisted of 344 eighth grade students in main districts of Konya, Turkey, chosen by simple random sampling. 181 of the students were male, while 163 were female.

Data Collection Tools

The Equation Test

The Equation Test was developed by the researcher. The test consisted of 15 open-ended questions. Three questions aimed to test the equation type ax=b (type 1), where the coefficients were integers; four questions aimed to test the equation type $\frac{a}{b}x = c$ (type 2), where the coefficients were rational numbers; four questions aimed to test the equation type a(x+b)+c=d (type 3), which involves parenthesis; and four questions aimed to test the rational equation $\frac{a}{bx \pm c} = \frac{d}{ex \pm f}$ (type 4). A rubric developed by the researcher was used to mark the answers. Accordingly, the rubric for the Equation Test was as follows:

Table 1. Rubric for the equation test

Score	*Explanation*
0	No or wrong solution and no or wrong result
1	Solution partially correct, wrong result
2	Solution correct, wrong result
3	Solution correct and correct result

The reliability analysis of the test conducted with 144 students indicated a Cronbach α reliability coefficient of 0.94. The equation test, which was developed following a pilot study, was administered to a sample of 344 students. The number of items for the equation types in the test varied. Therefore, absolute scores were calculated. This calculation was as follows:

Table 2. Absolute scores used for the analysis of the equation test

	No of Items	*Score Interval*	*Max. Score*	*Coefficient*	*Standardized Score*
Equation Type 1	3	0-3	0-9	*4/3	0-12
Equation Type 2	4	0-3	0-12	*1	0-12
Equation Type 3	4	0-3	0-12	*1	0-12
Equation Type 4	4	0-3	0-12	*1	0-12

The Proportional Reasoning Test

The Proportional Reasoning Test was developed by Akkuş and Duatepe-Paksu (2006) in order to assess the proportional reasoning skills of primary school students. It consists of three parts. The first part (PRT 1) involves eight items that require "finding the missing value and inverse proportion"; the second part (PRT 2) involves three open-ended items that require quantitative comparison; and the third part (PRT 3) involves five items that require qualitative comparison. A rubric which was also created by the test developers was used to mark the answers. The rubric, provided in Tables 3, 4 and 5 suggested different scores for each part of the test.

Table 3. Rubric for the first part of the proportional reasoning test

Score	*Explanations*
0	• No answer. • No indication for the existence of proportional reasoning. • An aggregate comparison of the data exists. • Random use of the data, numbers and operations.
1	• Only the result is provided. • Some indication for the existence of proportional reasoning (such as creating a proportion between wrong variables, creating a proportion using visual data). • Proportion type is not recognised.
2	• Proportional reasoning exists between the expected variables, but the result is not obtained. • Proportional reasoning exists between the expected variables, but the student made an operational error.
3	• Proportional reasoning required to solve the problem completely and correctly exists and the result is obtained.

Table 4. Rubric for the second part of the proportional reasoning test

Score	*Explanations*
0	• No answer. • Only the result is provided. • Proportion was created between wrong variables. • No indication for the existence of proportional reasoning. • An aggregate comparison of the data exists. • Random use of the data, numbers and operations.
1	• Right answer is obtained by either using or not using proportional reasoning skills between the expected variables, but the interpretation is wrong. • Right answer provided, but the explanation is insufficient.
2	• Proportional reasoning skill between the expected variables is presented, the right answer is provided, but the explanation is insufficient.
3	• Proportional reasoning skill between the expected variables exists, but the right answer is not obtained due to an operational error. • Even if the right answer was not provided, it is interpreted correctly based on the result obtained.
4	• Proportional reasoning skill required to obtain the right answer is well represented and correct interpretation is provided.

Table 5. Rubric for the third part of the proportional reasoning test

Score	*Explanations*
0	• No answer. • No indication for the existence of proportional reasoning. • Only the correct answer is chosen, no explanation provided.
1	• The result is obtained by using only one of the data given in the question and correct answer is chosen.
2	• Correct answer is chosen, wrong or incomplete explanation is provided using both of the data given in the question.
3	• The expected correct answer is given; the explanation is provided by using the expressions found in the question stem.
4	• The expected correct answer is given; the explanation is provided not with the expressions found in the question stem but with original sentences, the explanations were enriched with explanations like figures, drawings, and examples.

The Cronbach alpha internal consistency coefficient of the Proportional Reasoning Test calculated by Akkuş and Duatepe-Paksu (2006) was 0.86. This reliability coefficient calculated by Akkuş and Duatepe-Paksu (2006) was accepted for the current study and it was not recalculated by the researcher. Similar to the Equation Test, as the parts of the Proportional Reasoning Test consisted of varying number of items, absolute scores were calculated in order the standardise the scores. The calculations were carried out as follows:

Table 6. Absolute scores used in the proportional reasoning test

	No of Items	Score Interval	Max. Score	Coefficient	Standardised Scores
P.R.T.1	8 items	0-3	0-24	*1	0-24
P.R.T.2	3 items	0-4	0-12	*2	0-24
P.R.T.3	5 items	0-4	0-20	*1,2	0-24

Data Analysis

Descriptive statistics such as arithmetic means and standard error were calculated in order to determine and interpret students' performance in solving equations both in the proportional reasoning test and in general. In order to investigate the relationship between proportional reasoning and success in solving equations the Pearson Product Moments Correlation Coefficient was calculated. An F test was administered in order to find out whether students' success differed in terms of equation types and proportional reasoning question types. If such a difference existed, a Scheffe test was then run in order to determine which group created the variation. Moreover, simple linear regression analysis was conducted in order to identify the power of proportional reasoning to explain and predict success in solving equations. The analysis were carried out with SPSS 15.0 statistical software and then interpreted.

FINDINGS

Findings about the Equation Test

Descriptive statistics for the equation test are presented in Table 7.

Table 7. Descriptive statistics for the equation test

	N	$\bar{x}$	S.S
Equation Type 1	344	7.71	4.15
Equation Type 2	344	6.98	4.91
Equation Type 3	344	6.97	4.92
Equation Type 4	344	5.93	5.05

The figures in Table 7 indicate that eighth grade students were most successful at the first type equation (equations with integer coefficients) with a mean score of $\bar{x}$=7.71 out of 12. This was followed by the second type equation with a mean score of $\bar{x}$=6.98, then the third type with a mean score of $\bar{x}$=6.97 and finally the fourth type with a mean score of $\bar{x}$=5.93. Thus, the students were observed to be least successful at the fourth type of equations. One-way variance analysis was then calculated in order to test whether the variance in eighth grade students' mean scores was statistically significant. The results of this test are presented in Table 8.

Table 8. Variance analysis results for the equation test mean scores

Source of Variance	*Sum of Squares*	*Degree of Freedom*	*Mean of Squares*	*F*	*p*
Between Groups	551.576	3	183.859	8.06	.00
Within Groups	31297.715	1372	22.812		
Total	31849.290	1375			

* significant at α =0.05 level

According to Table 8, F value (F=8.060, p<0.05) was statistically significant at α =0.05 level. This finding indicated that the variance in the mean scores of 8th grade students for the four types of equations was statistically significant. A Scheffe test was further conducted in order to determine which equation type created this variance. The results are provided in Table 9.

Table 9. Variance in the mean scores in relation to the equation types

	Equation T1	*Equation T2*	*Equation T3*	*Equation T4*
Equation Type 1	-	.731	.739	1.777(*)
Equation Type 2	-.731	-	.008	1.046(*)
Equation Type 3	-.739	-.008	-	1.037(*)
Equation Type 4	-1.777(*)	-1.046(*)	-1.037(*)	-

Table 9 shows that while the differences between the mean scores of Type 1 equations, and Type 2 and 3 equations were not significant; the difference between the mean scores of Type 1 and Type 4 equations was significant. The figures in the table also suggest that the differences between the mean scores of Type 4 equations, and Type 2 and 3 equations were significant. However, the differences were not significant for the rest of the equations. Therefore, it is possible to conclude that the variation in the scores of eighth grade students in the equation test was created by the difference between Type 4 equations and the rest of the equations. In other words, eighth grade students were significantly more successful at Type 1, 2, and 3 equations than Type 4 equations.

Findings about the Proportional Reasoning Test

Descriptive statistics calculated for the proportional reasoning test are presented in Table 10.

Table 10. Descriptive statistics for the proportional reasoning test

	N	$\overline{x}$	*S.S*
Proportional Reasoning Test 1(PRT 1)	344	11.8	9.1
Proportional Reasoning Test 2(PRT 2)	344	9.0	8.9
Proportional Reasoning Test 3(PRT 3)	344	9.1	7.5

(Hereafter PRT 1 will be used for first part of proportional reasoning test, PRT 2 for second part of test and PRT 3 for third part of test)

As shown in Table 10, eighth grade primary school students were mot successful at the first dimension of the proportional reasoning test (items about finding the missing value and inverse proportion) with a mean score of $\overline{x}$=11.8. This was followed by the third dimension (items that required qualitative comparison) with a mean score of $\overline{x}$=9.1 and then the second dimension (items that required quantitative comparison). One-way variance analysis was carried out in order to identify whether students' mean scores for the three dimensions were significantly different. The results of the variance analysis are presented in Table 11.

Table 11. Variance analysis results of the proportional reasoning test scores

Source of Variance	*Sum of Squares*	*Degree of Freedom*	*Mean of Squares*	*F*	*p*
Between Groups	1713.134	2	856.567	11.68	.00
Within Groups	75442.101	1029	73.316		
Total	77155.235	1031			

Variance analysis results indicated that eighth grade primary school students' mean scores for the dimensions of finding the missing value-inverse proportion, quantitative comparison and qualitative comparison were significantly different $\left[F_{(2-1029)} = 11.683,\ \ p < 0.05\right]$. A Scheffé was administered in order to determine which dimension created the variance. The results are shown in Table 12.

Table 12. Variance in the mean scores of the dimensions of the proportional reasoning test

	PRT 1	*PRT 2*	*PRT 3*
PRT 1	-	2.78(*)	2.67(*)
PRT 2	-2.78(*)	-	-.11
PRT 3	-2.67(*)	.11	-

Table 12 shows that primary school eighth grade students' mean scores for the first dimension of the proportional reasoning test which required finding the missing value and doing inverse proportion was significantly different than the mean scores of the second dimension which required quantitative comparisons and the mean scores of the third dimension which required qualitative comparison. The difference was significant at α=0.05 level in favour of the first dimension. In other words, eighth grade students were more successful at the first dimension of the proportional reasoning test, i.e. finding the missing value and doing inverse proportion, than the other two dimensions. On the other hand, while students' mean scores for the second dimension (quantitative reasoning) was slightly lower than that of the third dimension (qualitative comparison); the difference between these two mean scores was not significant.

Singh (2000) reported similar results for proportional reasoning skills of ninth grade students in finding the missing value, quantitative reasoning and qualitative reasoning.

The Relationship between Proportional Reasoning Skills and Success in Solving Equations

In order to investigate the relationship between proportional reasoning skills and success in solving equations, Pearson Product Moments Correlation Coefficient was initially calculated using the scores obtained from the equation test and the proportional reasoning test. The coefficients are presented in Table 13.

Table 13. Pearson product moments correlation coefficients of eighth grade students' proportional reasoning test scores and equation test scores

	Total Equation	*E.T.1*	*E.T.2*	*E.T.3*	*E.T.4*	*Total P.R.T.*	*P.R. T.1*	*P.R.T. 2*	*P.R.T 3*
Total Equation	1	.86**	.90**	.94**	.92**	.83**	.85**	.69**	.58**
E.T.1	.86**	1	.78**	.81**	.75**	.73**	.76**	.62**	.48**
E.T.2	.90**	.78**	1	.82**	.81**	.78**	.77**	.68**	.56**
E.T.3	.94**	.81**	.82**	1	.89**	.79**	.79**	.68**	.57**
E.T.4	.92**	.75**	.81**	.89**	1	.84**	.84**	.72**	:59**
Total P.R.	.83**	.73**	.78**	.79**	.84**	1	.92**	.87**	.81**
P.R. T.1	.85**	.76**	.77**	.79**	.84**	.92**	1	.75**	.56**
P.R. T.2	.69**	.62**	:68**	.68**	.72**	.87**	.75**	1	.62**
P.R. T.3	.58**	.48**	.56**	.57**	.59**	.81**	.56**	.62**	1

As presented in Table 13, the correlation coefficient between the total score of the proportional reasoning test and the total score of the equation test was r=0.84. This value indicated a significant positive relationship at α=0.01 level between eighth grade students’ proportional reasoning skills and their success in solving equations. Therefore, it is possible to conclude that there was a strong relationship between eighth grade students’ proportional reasoning skills and their success in solving equations.

The correlation coefficients calculated between the total score of the equation test and the dimensions of the proportional reasoning test were respectively; r= 0.85 for the dimension of the proportional reasoning test which included questions about finding the missing value and doing inverse proportion; r=0.69 for the dimension which included quantitative comparison; and r=0.58 for the dimension which included qualitative comparison. All these coefficients indicated positive significant relationships at α=0.01 level, and it is possible to conclude that eighth grade students’ success in solving equations was most strongly related to the first dimension which included questions that required finding the missing value and doing inverse proportion. This was followed, respectively, by the second dimension which included quantitative comparison and the third dimension which included qualitative comparison.

The correlation coefficients calculated between eighth grade students’ scores of Type 1 equations (equations with integer coefficients), Type 2 equations (equations with rational coefficients), Type 3 equations (equations with parenthesis) and Type 4 equations (rational proportions); and the dimension of proportional reasoning test (PRT) which included questions that required finding the missing value and doing inverse proportion were respectively r_1=0.76, r_2=0.78, r_3=0.79, and r_4 =0.84; and the dimension which included quantitative comparison questions were r_1 =0.63, r_2 =0.69, r_3 =0.69, r_4 =0.72; and the dimension which included qualitative comparison questions were r_1=0.49, r_2 =0.57, r_3=0.57, r_4 =0.59. All these coefficients were significant at α=0.01 level. Eighth grade students’ success in solving equations for all of the type 1, type 2, type 3 and type 4 equations was most strongly related to the first dimension of the PRT which required finding the missing value-doing inverse proportion. This was followed, respectively, by the second dimension which included quantitative comparison questions and then the third dimension which included qualitative comparison.

As presented in the correlations table, the correlation coefficients between the total score of the equation test and Type 1 equation score (equations with integer coefficients) was 0.86; Type 2 equation score (equations with rational number coefficients) was 0.90; Type 3 equation score (equations with parenthesis) was 0.95; and Type 4 equation score (rational equations) was 0.92. All of these correlation coefficients were significant at α=0.01 level. The highest correlation obtained between the score of equations with parenthesis and the equation test total score (r=0.95) could be interpreted as an indication that equations with parenthesis were the main determinant equation type for the success levels of primary school eighth grade students' in solving equations.

The correlation coefficients between the total score of the proportional reasoning test and the first dimension of the test which required finding the missing value-doing inverse proportion was 0.92; between the total score and the second dimension which included quantitative comparison questions was 0.88; between the total score and the third dimension which included qualitative reasoning questions was 0.82. As implied by these values, the most important dimension that explained proportional reasoning skill was the first dimension which included items that required finding the missing value and doing inverse proportion.

The Strength of the Relationship between Proportional Reasoning and Success in Solving Equations

In order to determine the strength of the relationship between eighth grade students' proportional reasoning skills and their success in solving equations, a multiple regression analysis was conducted where proportional reasoning was the independent and success in solving equations was the dependent variable. The relationship between the two variables was checked against whether it was linear in terms of the total scores. The regression analysis was conducted after obtaining a linear relationship. The results of the analysis are presented in Table 14 and Table 15.

Table 14. The power of proportional reasoning skill to explain success in solving equations

Model	R	R^2	F
Proportional Reasoning	.861	.741	324.027*

* significant at $\alpha = 0.05$ level

The regression coefficient calculated by using success in solving equations as the dependent and proportional reasoning skill as the independent variable was a=0.86. This outcome indicated a strong and significant relationship. Accordingly, it is possible to conclude that 74% of the difference (variance) between students' equation test scores was created by the proportional reasoning skill used in different question types. The F value (324.027) indicated that proportional reasoning skill significantly explained success in solving equations ($p<0.05$).

The power of proportional reasoning skill to predict success in solving equations is provided in Table 15.

Table 15. The power of proportional reasoning skill to predict success in solving equations

Variable	*Non-Std. Beta*	*Std. Error*	*Std. Beta*	*t*	*p*
P.R.T.1	1.325	.079	.720	16.783	.000
P.R.T.2	.136	.085	.072	1.602	.110
P.R.T.3	.299	.080	.134	3.755	.000

According to the standardized regression coefficient (Beta), the predictive variables of success in solving equations were respectively proportional reasoning 1 (B= 0.720), proportional reasoning 3 (B= 0.134) and proportional reasoning 2 (B= 0.072).

An analysis of the t-test results conducted to test the significance of the regression coefficients indicated that, except the second dimension (quantitative reasoning), proportional reasoning required in the other two dimensions were significant predictors of success in solving equations. In other words, primary school eighth grade students' proportional reasoning skills in the question types which required finding the missing value-doing inverse proportion significantly predicted their success in solving equations.

DISCUSSION AND CONCLUSION

The findings of the current study indicated that primary school eighth grade students were most successful at solving equations with integer coefficients. This was followed, respectively, by equations with rational number coefficients; equations with parenthesis and rational equations. While there were no significant differences between primary school eighth grade students' mean scores of equations with integer coefficients, equations with rational coefficients and equations with parenthesis; their scores in these three types of equations were significantly different than their mean scores in solving rational equations. Therefore, eighth grade students were less successful at solving rational equations than solving other types of equations.

According to the descriptive statistics for the proportional reasoning test, primary school eighth grade students were most successful at the first dimension of the test which required finding the missing value and doing inverse proportion. This was respectively followed by the third dimension, which required qualitative reasoning and the second dimension which required quantitative reasoning. The variance between eighth grade students' scores in the different dimensions of proportional reasoning test was tested for significance. A significant difference was obtained between the first dimension (finding the missing value and doing inverse proportion); and the second (quantitative reasoning) and the third (qualitative reasoning) dimensions.

However, the difference between the second dimension and the third dimension was not statistically significant. The significant difference in mean scores found between the first dimension and the rest could be explained by teachers' frequent use of the cross-product algorithm in teaching ratios and proportions. Moreover, the difference could also be due to the emphasis on questions that require finding the missing value, rather than qualitative and quantitative comparisons, in teaching ratios and proportions. These results were in parallel with the findings of Singh (2000).

The correlation coefficients, which were calculated in order to identify the relationship between primary school eighth grade students' proportional reasoning skills and their success in solving equations, were relatively strong and significant (r=0.84). This indicated a strong relationship between students' success in solving equations and their proportional reasoning skills. The development of students' proportional reasoning can be regarded as the gateway to success in studying algebra (Cai and Sun, 2002). The strong relationship obtained in the current study supported this argument.

Furthermore, in terms of the total test scores of proportional reasoning, rational equations had the highest correlation among the equation types. This was followed, respectively, by equations with parenthesis, equations with rational coefficients and equations with integer coefficients. Strong correlation coefficients obtained with rational equations could be due to their structural similarity to direct proportion. Stronger relationship observed between success in solving equations and the first dimension (finding the missing value-inverse proportion) of the proportional reasoning test also reinforces this interpretation.

The first dimension of the proportional reasoning test (finding the missing value, inverse proportion) had the strongest value among correlations between the four types of equations. This was followed, respectively, by the second dimension (quantitative reasoning questions) and the third dimension (qualitative reasoning questions). It is worth noting that the strong relationship between students' success in question types that require finding the missing value-inverse proportion and their success in all types of equations could be due to several reasons, such as changes in proportional reasoning performance in relation to question types (Tourniaire, 1984); teachers' heavy reliance on cross-product algorithm and question types that require finding the missing value when teaching proportions; and the effects of students' previous encounters with certain question types on their success in problem solving (Baykul, 2006).

Although studies in relevant areas have yet to be carried out in Turkey, it is possible to conclude that this strong relationship observed in the current study could be due to high emphasis on finding the missing value and doing inverse proportions and less practice with question types that require quantitative and qualitative reasoning in the teaching of ratios and proportions.

The regression analysis indicated that in terms of total scores proportional reasoning skills strongly explained success levels in solving equations (R=0.74). According to the regression analysis, the most significant predictive variable for success in solving equations was the first dimension of the proportional reasoning test (finding the missing value-inverse proportion). This was followed by the third dimension (qualitative comparison) and then the second dimension (quantitative reasoning). As mentioned above, heavy emphasis on finding the missing value seemed to explain these findings. Therefore, it would be interesting to replicate the study with a sample of students who had adequately used all three types of proportional reasoning questions.

Students were observed to have lower scores in rational equations which had the strongest relationship with proportional reasoning skills. Scores were also low in questions that required qualitative and quantitative reasoning which could be significant in the development of proportional reasoning skills. These results revealed a necessity to practise these two questions types more in teaching.

In the teaching of proportions, cross-product algorithm should be introduced later in solving question types which require finding the missing value. Instead, strategies that highlight proportional reasoning skills should be foregrounded.

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